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Boundary S-matrix in a $(2,0)$ theory of AdS_3 supergravity

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Abstract: We will discuss two inequivalent generalizations of the standard $(2,0)$ supergravity action [1] to include gravitational Chern-Simons term. One is in the first order formalism where we treat ${\omega_M}^{ab}$ as independent and the other is in the second order formalism where ${\omega_M}^{ab}$ is determined in terms of other fields via a standard constraint equation. The two theories have different equations of motion and the solutions to the equations of motion of the first order theory spans only a subset of those of the second order theory. We will be interested in computing the boundary S-matrix, describing correlation functions in a dual conformal field theory, in this sector and hence we use the equations of motion coming out of the first order theory. We restrict ourselves to the gauge+fermionic sector of the theory to compute the boundary S-matrix. We will also look at the effect of higher derivative terms on the boundary S-matrix obtained using the standard supergravity action.

Keywords: Black Holes in String Theory, Extended Supersymmetry, AdS-CFT Correspondence.

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1. Introduction

Three dimensional AdS_3 supergravity has always been an interesting area of study since the discovery of BTZ black holes [2]. It has also played an important role in string theory since BTZ arises as a factor in the near horizon geometry of certain class of black holes in string theory [3]. The entropy of BTZ black holes has been computed in two derivative theory of gravity [4] as well as higher derivative theories of gravity [5 − 11]. The statistical analysis of the entropy [4 − 8, 10] exploits the asymptotic symmetry of BTZ black hole and AdS/CFT correspondence [12] whereas the gravity analysis in [9] and [11] applies Wald's formula in the presence of gravitational Chern-Simons term. The entropy of BTZ black holes in the presence of arbitrary higher derivative terms has also been computed in [13] from the first law of thermodynamics which also includes various aspects of statistical entropy. All the above analysis reproduce the same result for the entropy of BTZ black holes which has remarkable similarity with the Cardy formula for the degeneracy of states in the two dimensional conformal field theory.

Later, Kraus and Larsen argued using AdS/CFT correspondence that if the theory has extended supersymmetry then the entropy of such black holes is given completely in terms of the coefficient of the gravitational Chern-Simons term and the coefficient of the Chern-Simons term involving the R-Symmetry gauge field [6, 7]. A bulk interpretation of their result was understood in [14] for theories with (0,4) supersymmetry and higher. The main conclusion of [14] was that the boundary S-matrix does not get renormalized by the addition of higher derivative terms. Since the boundary S-matrix are the only perturbative observables in the bulk theory, their result implied that the bulk action even does not get renormalized in the bulk. This in turn implies the non-renormalization of the black hole entropy.

Later, in [15] Gupta and Sen came up with a result where the non-renormalization of the bulk action does not resort to the boundary S-matrix. They found that one can obtain a direct field redefinition in the bulk to absorb the higher derivative piece to bring the action to the standard form. The standard form of the action contains the Einstein-Hilbert piece with a negative cosmological constant term, gravitational Chern-Simons term and its supersymmetrization, Chern-Simons like term involving the gravitino. The standard form also has a Chern-Simons term involving the R-Symmetry gauge field if the theory has an extended supersymmetry. Such field redefinition does not affect the Chern-Simons terms but *a priori* it allows for the cosmological constant to change.[1] But the cosmological constant is determined in terms of the coefficient of the gravitational and gauge Chern-Simons term in a theory with extended supersymmetry. Hence, even the cosmological constant does not change by the field redefinitions.

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Since the results of “Kraus and Larsen” and “Gupta and Sen” holds for all theories of gravity with extended supersymmetry, we would expect that the boundary S-matrix does not get renormalized even for theories with lesser extended supersymmetry (*e.g.* theories with $(2,0)$ supersymmetry). The meaning of S-matrix not getting renormalized is however upto unitary transformation. Two S-matrices related by unitary transformations are the same and are related by redefinition of currents in the boundary. The field redefinition in the bulk induces this redefinition of currents in the boundary and hence the S-matrices calculated from two actions related by field redefinition in the bulk are the same upto unitary transformation. And since by the results of “Gupta and Sen” [15] restricted to $(2,0)$ theory, all $(2,0)$ actions are related by field redefinitions, this implies that the S-matrix with and without higher derivative terms are related by unitary transformations. This also implies the non-renormalization of central charges in the boundary CFT even for $(2,0)$ theories which in turn implies Kraus and Larsen’s non-renormalization of black hole entropy even for $(2,0)$ theories.

However, one might wonder that what sort of higher derivative terms do not alter the boundary S-matrix computed using the standard supergravity action[2] and what sort of higher derivative terms alter the standard boundary S-matrix upto unitary transformation. We shall be interested in addressing this issue in our paper.

For boundary S-matrix in a $(0,4)$ theory of supergravity involving SU(2) current correlators, this unitary transformation is exactly identity as observed in [14]. This observation however fails for a certain class of higher derivative terms in $(2,0)$ case. In order to distinguish such terms let us try to recap the results of [14].

A theory of supergravity with $(0,4)$ supersymmetry has a SU(2) R-symmetry and

[1]For a theory of pure gravity, the effect of higher derivative terms will be a renormalization in the bare parameters like cosmological constant. This phenomena has also been observed in [16, 17] but this situation changes drastically in a theory of extended supergravity because the higher derivative terms does not affect the coefficients of Chern-Simons terms and since the cosmological constant is related to the coefficient of these Chern-Simons terms, even the cosmological constant is not modified in a theory of extended supergravity with arbitrary higher derivative terms.

[2]We shall sometimes refer to the boundary S-matrix computed using the standard supergravity action as the standard boundary S-matrix.

a SU(2) gauge field A^a_M corresponding to that. This gauge field sources the SU(2) R-symmetry current in the boundary CFT. One then works with the standard Chern-Simons action S_0 and truncates to just the bosonic part. The gauge field equation of motion in this case becomes $F^a_{MN} = 0$. In order to obtain the correlation function involving this current, one first obtains a solution to the gauge field equation of motion $F^a_{MN} = 0$ with a boundary condition specified on A^a_z. After putting this solution in the action S_0 one obtains a functional $I[A^{(0)a}_z]$ of the boundary values $A^{(0)a}_z$. Then according to AdS/CFT correspondence $e^{-I[A^{(0)a}_z]}$ is interpreted as the partition function for calculating the correlation functions in the boundary [18, 19] .

One can then add higher derivative gauge invariant term to the action S_0 as

$$S = S_0 + \lambda \int d^3x F^a_{MN} K^{aMN}, \tag{1.1}$$

where F^a_{MN} is the gauge covariant field strength. K^{aMN} is some arbitrary term constructed out of field strengths, Riemann tensor etc. It is gauge covariant and hence it carries the gauge index a which is required to make the full action gauge invariant. Thus K^{aMN} should contain at least one power of F^a_{MN}. This implies that the additional terms in the action (1.1) contains at least two powers of F^a_{MN} as a result of which it does not alter the equation of motion $F^a_{MN} = 0$ which is also the equation of motion without the additional term. As a consequence of this the additional term vanishes on-shell and hence does not alter the boundary S-matrix.

The crux of the above observation is that additional higher derivative terms do not change the original equations of motion and hence vanish on-shell. But on the contrary for $(2, 0)$ theories, as we will see later, there can be two sets of higher derivative terms

1. Terms constructed out of field strengths covariant with respect to supersymmetry transformations under which the standard action remains invariant.[3] Such terms will be the basic building blocks for constructing higher derivative terms which renders the full action to be invariant under the original supersymmetry transformation laws. We will see in the beginning of section 4 that such terms do not change the original equations of motion, and as a consequence of this, vanish on-shell and hence does not alter the boundary S-matrix. The unitary transformation, discussed earlier, in this case will be exactly identity.

2. However, there can be other higher derivative terms which can render the full action to be invariant under a set of supersymmetry transformation laws different from the original supersymmetry transformation laws.[4] We will see towards the end of section 4 that such terms can potentially change the equations of motion of the standard supergravity action and hence do not vanish on-shell, and as a result of this, can alter the boundary S-matrix calculated from the standard action. However,

[3] We will give an explicit definition and derivation of field strengths covariant with respect to the original supersymmetry transformation rules in section 3.

[4] We will sometimes refer to the original supersymmetry transformation laws as the “standard supersymmetry transformation laws”.

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such terms can be removed by a redefinition of the fields[5] and hence, as discussed before, the change in S-matrix will be a non-trivial unitary transformation.

For the sake of completeness, we will compute, in the end, the boundary S-matrix involving the correlation function of the boundary currents from the standard action S_0. But in this case we cannot just restrict ourselves to the gauge sector. This is because the gauge Chern-Simons action for a U(1) gauge field is

$$S_{\texttt{gauge}} = -\frac{a_L}{2}\int d^3x\, \epsilon^{MNP} A_M \partial_N A_P \tag{1.2}$$

In this case we can arbitrarily scale A_M to change the coefficient before it. This was not the case for $(0,4)$ theory in [14] because apart from the $A \wedge dA$ term in the action there was a $A \wedge A \wedge A$ term which forbids the arbitrary scaling of coefficient before the gauge Chern-Simons term. So, for $(2,0)$ theory we need to have in the action a term which contains a power of A different from two. And indeed there is such a term in the action which is the coupling of gravitino with the gauge field. Thus we need to work with gauge and fermionic sector simultaneously and calculate correlation function involving the R-symmetry current $J(z)$ and supersymmetry currents $G^{(+)}(z), G^{(-)}(z)$ from the standard action S_0.

We organize the paper as follows:

1. In section 2 we will discuss the generalization of standard $\mathcal{N} = (2,0)$ supergravity action [1] to include gravitational Chern-Simons term. We will see that we can have two inequivalent generalizations and we will discuss in great detail the differences between them.

2. In section 3 we will define and obtain the field strengths and Riemann tensor covariant with respect to standard supersymmetry transformation laws. These will form the building blocks of constructing supersymmetric invariant higher derivative terms respecting the standard supersymmetry transformation laws.

3. In section 4 we discuss the implication of addition of higher derivative terms respecting supersymmetry on the boundary S-matrix which shall arise from the standard supergravity action. We will discuss two sets of higher derivative terms. First we will consider terms which are constructed out of the super-covariant field strengths discussed in section 3 and we will see that they do not alter the standard boundary S-matrix. In the end we will consider more general terms which alter the standard boundary S-matrix by a non-trivial unitary transformation.

4. To conclude, we will compute the boundary S-matrix describing correlation functions involving R-symmetry current $J(z)$ and supersymmetry current $G^{(+)}(z)$, $G^{(-)}(z)$ from the standard supergravity action in section 5. In particular, we calculate the correlators $\langle J(z)J(w)\rangle$, $\langle G^+(z)G^-(w)\rangle$ and $\langle J(z)G^+(v)G^-(w)\rangle$. We will see that the results match with the conformal field theory results.

[5]Field redefinition does not change the AdA and $\bar{\psi}A\psi$ term but it can potentially change the $\bar{\psi}d\psi$ term. However U(1) gauge invariance fixes the relative coefficient of the $\bar{\psi}d\psi$ and $\bar{\psi}A\psi$ term and hence even this coefficient is also not changed.

2. $\mathcal{N} = (2,0)$ supergravity action

We shall be working with a generalization of standard $(2,0)$ supergravity [1] to include gravitational Chern-Simons term.[6] Such a generalization has been obtained in [20] for the simple case of $\mathcal{N} = 1$ supersymmetry. We shall be interested in the case having $(2,0)$ supersymmetry.

Before trying to obtain a $(2,0)$ supersymmetric generalization of topologically massive supergravity with a cosmological constant — which we shall refer to as cosmological topologically massive supergravity, let us first look at the standard $(2,0)$ supergravity without gravitational Chern-Simons term [1].[7] The field contents are the vierbeins ${e_M}^a$, U(1) gauge field A_μ, and a complex Rarita-Schwinger field ψ_M. The action written in terms of the fields e^a_M, ψ_M, $\bar{\psi}_M$, A_μ is

$$S_0 = \int d^3x \left[eR + 2m^2 e - \frac{1}{2m}\epsilon^{MNP} A_M \partial_N A_P + \frac{i}{4m}\epsilon^{MNP}\left(\bar{\psi}_M(\mathcal{D}_N\psi_P) - (\mathcal{D}_N\bar{\psi}_M)\psi_P\right)\right] \tag{2.1}$$

We follow the following convention for Riemann tensor, Ricci tensor and Ricci scalar

$$\begin{aligned} {R_{MN}}^{ab} &\equiv 2\partial_{[M}{\omega_{N]}}^{ab} + 2{\omega_{[M}}^{ac}{\omega_{N]}}^{db}\eta_{cd}, \\ {R_M}^a &\equiv {e^N}_b {R_{MN}}^{ab}, \\ R &\equiv {e^M}_a {e^N}_b {R_{MN}}^{ab}, \end{aligned} \tag{2.2}$$

${\omega_M}^{ab}$ is the spin-connection and is determined in terms of ${e_M}^a$ and ψ_M from the equation

$$\partial_{[N}{e_{P]}}^a + {\omega_{[N}}^{ab} e_{P]b} = -\frac{i}{8m}\bar{\psi}_{[N}\gamma^a\psi_{P]} \tag{2.3}$$

and e is the determinant of ${e_M}^a$ given by[8]

$$e = \det({e_M}^a) = \frac{1}{6}\epsilon^{MNP}\varepsilon_{abc}{e_M}^a{e_N}^b{e_P}^c, \quad \epsilon^{012} = 1 \quad \varepsilon_{\hat{0}\hat{1}\hat{2}} = 1 \tag{2.4}$$

$\mathcal{D}_M\psi_N$ and $\mathcal{D}_M\bar{\psi}_N$ are defined as

$$\begin{aligned} \mathcal{D}_M\psi_N &= \partial_M\psi_N - \frac{1}{2}{B_M}^a\gamma_a\psi_N + \frac{i}{2}A_M\psi_N, \\ \mathcal{D}_M\bar{\psi}_N &= \partial_M\bar{\psi}_N + \frac{1}{2}{B_M}^a\bar{\psi}_N\gamma_a - \frac{i}{2}A_M\bar{\psi}_N, \end{aligned} \tag{2.5}$$

${B_M}^a$ is given by

$${B_M}^a = \frac{1}{2}\varepsilon^{abc}\omega_{Mbc} - m{e_M}^a \tag{2.6}$$

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[6] Such a theory is also known as the cosmological topological massive supergravity because of the presence of a massive excitation.

[7] We will often refer to this as the cosmological topological massless supergravity because of the absence of a massive excitation.

[8] We denote the curved space index M by $0, 1, 2$ and flat space index a by $\hat{0}, \hat{1}, \hat{2}$. The epsilon symbol with curved space index is denoted by ϵ and the epsilon symbol involving flat index is denoted by ε. The index in both cases is raised and lowered using η_{MN} and η_{ab} and we follow the mostly positive convention i.e $\eta = diag(-1, 1, 1)$.

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γ_a satisfies the algebra

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} \quad [\gamma_a, \gamma_b] = -2\varepsilon_{abc}\gamma^c \tag{2.7}$$

We can write the gravity part of the action (2.1) in a pure Chern-Simons form by defining the gauge field ${B'_M}^a$ in addition to ${B_M}^a$ as:

$${B'_M}^a = \frac{1}{2}\varepsilon^{abc}\omega_{Mbc} + m{e_M}^a \tag{2.8}$$

Then the action (2.1) takes the following form[9] [21 – 23]:

$$\begin{aligned} S_0 = & -\frac{1}{m}\int d^3x\ \epsilon^{MNP}\left[\frac{1}{2}{B_M}^a\partial_N{B_P}^b\eta_{ab} + \frac{1}{6}\varepsilon_{abc}{B_M}^a{B_N}^b{B_P}^c\right] \\ & + \frac{1}{m}\int d^3x\ \epsilon^{MNP}\left[\frac{1}{2}{B'_M}^a\partial_N{B'_P}^b\eta_{ab} + \frac{1}{6}\varepsilon_{abc}{B'_M}^a{B'_N}^b{B'_P}^c\right] \\ & - \frac{1}{2m}\int d^3x\ \epsilon^{MNP}A_M\partial_N A_P \\ & + \frac{i}{4m}\int d^3x\ \epsilon^{MNP}\left(\bar{\psi}_M(\mathcal{D}_N\psi_P) - (\mathcal{D}_N\bar{\psi}_M)\psi_P\right) \end{aligned} \tag{2.9}$$

Note that in the above action, ${B_M}^a$ and ${B'_M}^a$ are not independent. Hence, this form of the action is not very useful. But we will see later that we can have a different formulation where we treat ${\omega_M}^{ab}$ as an independent field and hence ${B_M}^a$ and ${B'_M}^a$ becomes independent. In that case, the standard supergravity action in the above Chern-Simons form (2.9) is useful. However, (2.9) form of the action in the present formulation is useful in some context like looking at the supersymmetry invariance etc. But whenever we use the above form of the action in the present formulation, we should always keep in mind that ${B_M}^a$ and ${B'_M}^a$ are not independent.

The action (2.1), (2.9) is invariant under the supersymmetry transformations

$$\begin{aligned} \delta_\epsilon^{(Q)} e_M^a &= -\frac{i}{8m}\left(\bar{\epsilon}\gamma^a\psi_P - \bar{\psi}_P\gamma^a\epsilon\right) \\ \delta_\epsilon^{(Q)} A_P &= \frac{1}{4}\left(\bar{\epsilon}\psi_P - \bar{\psi}_P\epsilon\right), \\ \delta_\epsilon^{(Q)}\psi_P &= \mathcal{D}_P\epsilon, \\ \delta_\epsilon^{(Q)}\bar{\psi}_P &= \mathcal{D}_P\bar{\epsilon}, \end{aligned} \tag{2.10}$$

Using (2.3) one can deduce the following supersymmetry transformation on the dependent gauge field ${\omega_M}^{ab}$

$$\begin{aligned} \delta_\epsilon^{(Q)}{\omega_P}^{ab} &= -\frac{i}{8}\varepsilon^{abc}\left(\bar{\epsilon}\gamma_c\psi_P - \bar{\psi}_P\gamma_c\epsilon\right) \\ &\quad + \frac{i}{16m}\left[\bar{\epsilon}\gamma_P G^{ab} - \bar{G}^{ab}\gamma_P\epsilon\right] \\ &\quad + \frac{i}{8m}\left[\bar{\epsilon}\gamma^{[a}{G_P}^{b]} - {\bar{G}_P}{}^{[b}\gamma^{a]}\epsilon\right] \\ &\equiv -\frac{i}{8}\varepsilon^{abc}\left(\bar{\epsilon}\gamma_c\psi_P - \bar{\psi}_P\gamma_c\epsilon\right) + {C_P}^{ab}(\epsilon, G) \end{aligned} \tag{2.11}$$

[9]One can check by expanding ${B'_M}^a$ and ${B_M}^a$ in the action (2.9) in terms of ω_M^{ab} and e_M^a that one indeed gets back the action (2.1).

Where G_{MN} is the field strength associated with the Rarita-Schwinger field ψ_M defined in (2.13). The last line in the above equation defines $C_P{}^{ab}(\epsilon, G)$. The supersymmetry transformations of the fields $B_M{}^a$ and $B'_M{}^a$ defined in (2.6), (2.8) takes the form

$$\begin{aligned}
\delta_\epsilon^{(Q)} B_P{}^a &= \frac{i}{4}\left(\bar{\epsilon}\gamma^a\psi_P - \bar{\psi}_P\gamma^a\epsilon\right) + \frac{1}{2}\varepsilon^{abc}C_{Pbc}, \\
\delta_\epsilon^{(Q)} B'_P{}^a &= \frac{1}{2}\varepsilon^{abc}C_{Pbc}
\end{aligned} \tag{2.12}$$

The equation of motion derived from the action (2.1) is

$$\begin{aligned}
\mathcal{R}_a{}^M - \frac{1}{2}\mathcal{R}e_a{}^M - m^2 e_a{}^M &= 0, \\
\widehat{F}_{MN} \equiv 2\partial_{[M}A_{N]} - \frac{1}{2}\bar{\psi}_{[M}\psi_{N]} &= 0, \\
G_{MN} \equiv 2\mathcal{D}_{[M}\psi_{N]} &= 0 \\
\bar{G}_{MN} \equiv 2\mathcal{D}_{[M}\bar{\psi}_{N]} &= 0
\end{aligned} \tag{2.13}$$

where

$$\begin{aligned}
\mathcal{R}_{MN}{}^{ab} &= R_{MN}{}^{ab} + \frac{i}{4}\varepsilon^{abc}\bar{\psi}_{[M}\gamma_c\psi_{P]}, \\
\mathcal{R}_M{}^a &= e_b{}^N\mathcal{R}_{MN}{}^{ab}, \\
\mathcal{R} &= e_a{}^M\mathcal{R}_M{}^a,
\end{aligned} \tag{2.14}$$

$\widehat{F}_{MN}$ and G_{MN} as we will see later are supersymmetric covariant field strengths for the gauge field A_M and Rarita -Scwhinger field ψ_M respectively and $\mathcal{R}_{MN}{}^{ab}$ is the supersymmetric covariant Riemann tensor modulo some terms proportional to G_{MN}. It is straightforward to check from (2.13) that $\mathcal{R}_{NP}$ satisfies

$$D_M\mathcal{R}_{NP} = 0 \tag{2.15}$$

Where D_M is the usual covariant derivative defined using the torsion free connections. The field equation corresponding to $\mathcal{R}_{MN}{}^{ab}$ can also be written in a more convenient way as

$$\begin{aligned}
\widehat{\mathcal{R}}_M{}^a &= 0 \\
\texttt{Where} \qquad \widehat{\mathcal{R}}_{MN}{}^{ab} &= \mathcal{R}_{MN}{}^{ab} + 2m^2 e_{[M}{}^a e_{N]}{}^b
\end{aligned} \tag{2.16}$$

In the above analysis of the theory of supergravity without the gravitational Chern-Simons term, we treated $\omega_M{}^{ab}$ as defined through the equation (2.3) and the only independent fields in the theory were $e_M{}^a$, ψ_M and A_M. This formulation is known in the literature as the "second order formulation" since it gives second order equations in the gravity sector. We will also resort to the supersymmetry transformations (2.11) and (2.12) for $\omega_M{}^{ab}$, $B_M{}^a$ and $B'_M{}^a$ as the "second order supersymmetry transformations".

In a different formulation we can treat $\omega_M{}^{ab}$ as an independent field in (2.1). The variation of the action with respect to this field will give rise to the constraint equation (2.3) and the variation with respect to other fields will give rise to the other equations in (2.13).

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All these equations are first order since we are treating ${\omega_M}^{ab}$ as independent. This formulation is known as the "first order formulation". In this formulation ${B_M}^a$ and ${B'_M}^a$ are independent and, as discussed before, the Chern-Simons form of the action (2.9) is a useful form to use.

We now discuss the supersymmetry invariance of the action under supersymmetry transformation independently defined for ${\omega_M}^{ab}$

$$\delta_\epsilon^{(Q)} {\omega_P}^{ab} = -\frac{i}{8}\varepsilon^{abc}\left(\bar{\epsilon}\gamma_c\psi_P - \bar{\psi_P}\gamma_c\epsilon\right) \tag{2.17}$$

This also implies the following supersymmetry transformation for the fields ${B_m}^a$ and ${B'_M}^a$

$$\begin{aligned}\delta_\epsilon^{(Q)} {B_P}^a &= \frac{i}{4}\left(\bar{\epsilon}\gamma^a\psi_P - \bar{\psi_P}\gamma^a\epsilon\right)\\ \delta_\epsilon^{(Q)} {B'_P}^a &= 0\end{aligned} \tag{2.18}$$

It is quite easy to check that the action (2.1), (2.9) is invariant with respect to this transformation along with (2.10) when we treat ${\omega_M}^{ab}$ as an independent gauge field. We will refer to the supersymmetry transformations (2.17) and (2.18) as the "first order supersymmetry transformations". From (2.18) we see that the supersymmetry transformation of ${B'_M}^a$ vanishes and this can be interpreted as belonging to the right sector of the theory where we do not have any supersymmetry whereas the rest of the fields (${B_M}^a$, ψ_M, A_M) belongs to the left sector which has $\mathcal{N} = 2$ supersymmetry. This justifies the $(2, 0)$ nature of the theory.

This theory is supersymmetric in both the formulations. Both the formulations also give rise to the same equations of motion. In particular, the ${\omega_M}^{ab}$ equation of motion in the first order formulation is the same as the equation that determines ${\omega_M}^{ab}$ in terms of ${e_M}^a$ and ψ_M in the second order formulation so that we can still interprete ${\omega_M}^{ab}$ as the spin connection.

Let us now try to make appropriate changes in (2.9) to obtain the gravitational Chern-Simons term and its supersymmetric generalization. Following [23, 14], we make the following changes

$$\begin{aligned}\mathcal{S}_0 &= -a_L\int d^3x\ \epsilon^{MNP}\left[\frac{1}{2}{B_M}^a\partial_N {B_P}^b\eta_{ab} + \frac{1}{6}\varepsilon_{abc}{B_M}^a{B_N}^b{B_P}^c\right]\\ &\quad +a_R\int d^3x\ \epsilon^{MNP}\left[\frac{1}{2}{B'_M}^a\partial_N {B'_P}^b\eta_{ab} + \frac{1}{6}\varepsilon_{abc}{B'_M}^a{B'_N}^b{B'_P}^c\right]\\ &\quad -\frac{a_L}{2}\int d^3x\ \epsilon^{MNP}A_M\partial_N A_P\\ &\quad +\frac{ia_L}{4}\int d^3x\ \epsilon^{MNP}\left(\bar{\psi}_M(\mathcal{D}_N\psi_P) - (\mathcal{D}_N\bar{\psi}_M)\psi_P\right)\\ a_L &= K + \frac{1}{m}\\ a_R &= -K + \frac{1}{m}\end{aligned} \tag{2.19}$$

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After replacing $B_M{}^a$ and $B'_M{}^a$ from (2.6) and (2.8) one can see that one indeed gets the gravitational Chern-Simons term

$$\begin{aligned}
\mathcal{S}_0 &= \int d^3x \left[eR + 2m^2 e - \frac{a_L}{2}\epsilon^{MNP} A_M \partial_N A_P + \frac{i}{4} a_L \epsilon^{MNP}\left(\bar\psi_M(\mathcal{D}_N\psi_P) - (\mathcal{D}_N\bar\psi_M)\psi_P\right)\right] \\
&\quad - K\int d^3x\ \epsilon^{MNP}\left[\left(\frac{1}{2}\omega_{Mcd}\partial_N\omega_P{}^{dc} + \frac{1}{3}\omega_{Mbc}\omega_N{}^{cd}\omega_{Pd}{}^b\right) + m^2 e_M{}^a\left(\partial_N e_P{}^a + \omega_{Nac}e_P{}^c\right)\right], \\
a_L &= K + \frac{1}{m}, \qquad (2.20)
\end{aligned}$$

In the first order formalism, when $\omega_M{}^{ab}$ is treated as an independent field, the first order supersymmetry transformation (2.18) and (2.10) of the left sector which comprises of $B_M{}^a$, ψ_M and A_M do not talk with the right sector comprising of $B'_M{}^a$. In the above action we have just changed the relative coefficients between the left and right sector fields without changing the relative coefficients between the fields of a particular sector. We therefore expect the above action to be invariant under the "first order" supersymmetry variations (2.17), (2.18), (2.10). We find that the action (2.19), (2.20) is indeed invariant under the first order supersymmetry transformations (2.17), (2.18), (2.10). In order to obtain the equation of motion we first vary the action (2.20)

$$\begin{aligned}
\delta\mathcal{S}_0 &= 2\int d^3x\ \epsilon^{MNP}\delta\omega_M{}^c\left[\partial_N e_{Pc} + \omega_{Nc}{}^d e_{Pd} + \frac{i}{8m}\bar\psi_N\gamma_c\psi_P\right] \\
&\quad - K\int d^3x\ \delta\omega_M{}^c\ e\left[\mathcal{R}e^M{}_c + 2m^2 e^M{}_c - 2\mathcal{R}^M{}_c\right] \\
&\quad + \int d^3x\ \delta e_M{}^a\ e\left[\mathcal{R}e^M{}_a + 2m^2 e^M{}_a - 2\mathcal{R}^M{}_a\right] \\
&\quad - 2Km^2\int d^3x\ \delta e_M{}^a\epsilon^{MNP}\left[\partial_N e_{Pa} + \omega_{Na}{}^d e_{Pd} + \frac{i}{8m}\bar\psi_N\gamma_a\psi_P\right] \\
&\quad - a_L\int d^3x\delta A_M\epsilon^{MNP}\left(\partial_N A_P - \frac{1}{4}\bar\psi_N\psi_P\right) + \frac{ia_L}{2}\int d^3x\delta\bar\psi_M\epsilon^{MNP}\mathcal{D}_N\psi_P \\
&\quad + \frac{ia_L}{2}\int d^3x\ \epsilon^{MNP}\mathcal{D}_N\bar\psi_P\delta\psi_M \qquad (2.21)
\end{aligned}$$

In the first order formulation when we are treating $\omega_M{}^{ab}$ as an independent field, we should set all the variations independently to zero to get the equations of motion. We get

$$\begin{aligned}
2\epsilon^{MNP}\left[\partial_N e_{Pc} + \omega_{Nc}{}^d e_{Pd} + \frac{i}{8m}\bar\psi_N\gamma_c\psi_P\right] + \\
-Ke\left[\mathcal{R}e^M{}_c + 2m^2 e^M{}_c - 2\mathcal{R}^M{}_c\right] &= 0 \\
-2Km^2\epsilon^{MNP}\left[\partial_N e_{Pa} + \omega_{Na}{}^d e_{Pd} + \frac{i}{8m}\bar\psi_N\gamma_a\psi_P\right] + \\
+e\left[\mathcal{R}e^M{}_a + 2m^2 e^M{}_a - 2\mathcal{R}^M{}_a\right] &= 0 \\
F_{MN} \equiv 2\partial_{[M}A_{N]} &= \frac{1}{2}\bar\psi_{[M}\psi_{N]}, \\
G_{MN} \equiv 2\mathcal{D}_{[M}\psi_{N]} &= 0 \qquad (2.22)
\end{aligned}$$

When $Km \neq \pm 1$ we can take linear combinations of first two equations and we get

$$\partial_{[N} e_{P]}{}^a + \omega_{[N}{}^{ab} e_{p]b} = -\frac{i}{8m} \bar{\psi}_{[N} \gamma^a \psi_{P]},$$
$$\mathcal{R}_a{}^M - \frac{1}{2} \mathcal{R} e_a{}^M - m^2 e_a{}^M = 0, \tag{2.23}$$

We thus see that when $Km \neq \pm 1$ we get the same equation that we got in our analysis of the supergravity theory without gravitational Chern-Simons term. However, when $Km = \pm 1$ the first two equations of (2.22) are degenerate and instead of two there is one equation governing $e_M{}^a$ and $\omega_M{}^{ab}$. This can also be seen from the action (2.19) written in terms of $B_M{}^a$ and $B'_M{}^a$. When $Km = \pm 1$ either a_L or a_R vanishes. Then we have either $B_M{}^a$ or $B'_M{}^a$ present in the action describing the gravity sector. Thus the degree of freedom required for a theory of gravity is reduced and we cannot have a theory of gravity. Hence, the first order formulation fails for $Km = \pm 1$.[10] However, at a generic point not satisfying $Km = \pm 1$ we get a perfectly sensible first order theory which is supersymmteric and gives the equations of motion which coincides with the equation of motion derived for the theory of supergravity without gravitational Chern-Simons term.

We cannot add higher derivative terms to the supergravity action in the first order formulation. The equation for $\omega_M{}^{ab}$ will become dynamical and there cannot be simple algebraic dependence of $\omega_M{}^{ab}$ on $e_M{}^a$ and ψ_M as in (2.3) and hence we cannot interpret $\omega_M{}^{ab}$ as the spin connection. Addition of higher derivative terms will require us to go the second order picture.

In the second order formulation we work with $e_M{}^a$, $\omega_M{}^{ab}$, ψ_M and A_M with $\omega_M{}^{ab}$ determined in terms of the other fields through (2.3). Substituting $\omega_M{}^{ab}$ by the constraint equation (2.3) in (2.20) gives rise to the action

$$\begin{aligned}
\mathcal{S}_0 &= \int d^3x \left[eR + 2m^2 e - \frac{a_L}{2} \epsilon^{MNP} A_M \partial_N A_P + \frac{i}{4} a_L \epsilon^{MNP} \left(\bar{\psi}_M (\mathcal{D}_N \psi_P) - (\mathcal{D}_N \bar{\psi}_M) \psi_P \right) \right] \\
&\quad - K \int d^3x\ \epsilon^{MNP} \left[\left(\frac{1}{2} \omega_{Mcd} \partial_N \omega_P{}^{dc} + \frac{1}{3} \omega_{Mbc} \omega_N{}^{cd} \omega_{Pd}{}^b \right) - \frac{im}{8} e_M{}^a \bar{\psi}_N \gamma_a \psi_P \right], \\
a_L &= K + \frac{1}{m},
\end{aligned} \tag{2.24}$$

However, the action (2.19) or (2.24) is not invariant under the "second order supersymmetry transformations" (2.10), (2.11), (2.12). Varying the action (2.19) with respect to these transformations, we get

$$\delta_\epsilon^{(Q)} \mathcal{S}_0 = \frac{K}{2} \int\ d^3x\ \epsilon^{MNP} C_{Mbc}(\epsilon, G)\ \widehat{\mathcal{R}}_{NP}{}^{bc} \tag{2.25}$$

This suggests that we should add to the action (2.19) or (2.24), terms constructed out of the field strengths $(\widehat{F}_{MN}, G_{MN}, \widehat{\mathcal{R}}_{NP}{}^{bc})$ such that the variation of such terms exactly cancel the variation of $\mathcal{S}_0$ obtained in (2.25). We will not require what exactly these terms are, but we will certainly need the fact that such terms are constructed out of the above mentioned field

[10]Such a phenomena has also been observed in a three dimensional theory of pure gravity in [13].

strengths and not the gauge fields. So these will fall into the category of "higher derivative terms" whose effect we will see later. Thus the complete cosmological topologically massive action of gravity invariant under the supersymmetry transformations (2.10)–(2.12) is

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_1 \left[\widehat{F}_{MN}, G_{MN}, \widehat{\mathcal{R}}_{NP}{}^{bc}\right] \tag{2.26}$$

Since $\mathcal{S}_1$ is higher in derivatives, we can use the results of [15] to argue that this term can be removed by an explicit field redefinition. In that case we suspect that $\mathcal{S}_0$ can still be supersymmetric even in the second order formulation by appropriately changing the transformation rules (2.10)–(2.12). Since we have not been able to find the modified transformation rules, we would rather keep the existing transformation rules and allow for the presence of $\mathcal{S}_1$ in the full supergravity action.

In order to get the equations of motion constructed out of $\mathcal{S}_0$ in the second order formulation we can use the variation (2.21), but keeping in mind that $\delta\omega_M{}^{ab}$ is not independent and should be determined in terms of $\delta e_M{}^a$ and $\delta\psi_M$ from the constraint equation (2.3). After using the constraint (2.3), the variation (2.21) takes the form

$$\begin{aligned}\delta\mathcal{S}_0 = & -K\int d^3x\ \delta\omega_M{}^c\ e\left[\mathcal{R}e^M{}_c + 2m^2 e^M{}_c - 2\mathcal{R}^M{}_c\right] \\ & + \int d^3x\ \delta e_M{}^a e\left[\mathcal{R}e^M{}_a + 2m^2 e^M{}_a - 2\mathcal{R}^M{}_a\right] \\ & - a_L\int d^3x \delta A_M \epsilon^{MNP}\left(\partial_N A_P - \frac{1}{4}\bar{\psi}_N\psi_P\right) \\ & + \frac{ia_L}{2}\int d^3x \delta\bar{\psi}_M \epsilon^{MNP}\mathcal{D}_N\psi_P \\ & + \frac{ia_L}{2}\int d^3x\ \epsilon^{MNP}\mathcal{D}_N\bar{\psi}_P\delta\psi_M\end{aligned} \tag{2.27}$$

To obtain the complete equation of motion systematically we should first express $\delta\omega_M{}^{ab}$ in terms of $\delta e_M{}^a$ and $\delta\psi_M$ and put it in the above variation. Then collect all the terms proportional to the variations of $e_M{}^a$, ψ_M and A_M and set them to zero. This will certainly not change the A_M equations because $\delta\omega_M{}^{ab}$ does not contain δA_M but it will certainly change the $e_M{}^a$ and ψ_M equations of motion. The equation, in general will be quite complicated involving the Cotton tensor [24, 25] and Cottino-vector spinor [26]. However, one can easily check that the equation of motion (2.13) obtained for the topologically massless case (which is also the equation of motion of $\mathcal{S}_0$ in the first order formulation) still extremizes the variation of $\mathcal{S}_0$ in the second order formulation and hence gives a sector of solution for the theory with gravitational Chern-Simons term in the second order formulation. This is contrary to the case of supergravity without gravitational Chern-Simons term where both the "first order" and "second order" formulation give rise to the same equations of motion. In the presence of gravitational Chern-Simons term, we saw that in the "first order" formulation the equations of motion are the same as that of the theory

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without gravitational Chern-Simons term. Whereas, in the "second order" formulation the equations get modified and the earlier equations span only a part of the full spectrum.[11]

We see two major differences in the first order and second order formulation of a theory of $(2,0)$ supergravity with gravitational Chern-Simons term. One is in the supersymmetry invariance of $\mathcal{S}_0$. The other is in the equations of motion spanned by both of them. Such differences in both the formulations in a theory of topologically massive gravity (TMG) and topologically massive electrodynamics (TME), as far as equations of motion is concerned, has been previously observed in [27]. However, later in [28] it was argued that the definition of the "first order theory" in [27] is not the correct first order formulation of the "second order theory" because both of them give different equations of motion. The correct first order formulation will include several other non-dynamical fields. They found the additional fields required for the case of TME. Finding the additional fields required for the correct first order formulation of a theory of gravity in the presence of gravitational Chern-Simons term in general will be quite involved and as per our knowledge there is no such formulation yet.

However, we will keep our definition of first order and second order formalism without calling one as the equivalent of the other. As we have seen before, the supergravity spectrum spanned by our first order theory belongs to a subset of the second order theory. We will be interested in the correlation functions of conformal field theory operators dual to this sector and hence we will use the equations of motion (2.13) in our analysis of boundary S-matrix.

3. Supersymmetric covariant field strengths

In this section we will try to covariantize the various field strengths (R_{MN}^{ab}, F_{MN}, G_{MN}, $\bar{G}_{MN}$) with respect to the standard second order supersymmetry transformations (2.10), (2.11), (2.12) considered in the previous section. First let us try to understand what do we mean by field strengths covariant with respect to supersymmetry. Normally the field strengths, *for example* Riemann tensor ${R_{MN}}^{ab}$ or gauge field strength $F_{MN} \equiv 2\partial_{[M}A_{N]}$, are not covariant with respect to supersymmetry i.e. under standard supersymmetry transformations (2.10), (2.11), (2.12) the above mentioned field strengths will give rise to terms proportional to partial derivatives of the supersymmetry transformation parameter. Thus we have to add certain terms to the above mentioned field strengths so that this non-covariant behavior gets canceled and we get perfectly covariant field strengths whose supersymmetry transformation gives rise to terms containing the supersymmetry transformation parameter and other supersymmetric covariant field strengths. As a result of this, if we add any term constructed out of these field strengths to the action, under supersymmetry it will give rise to terms involving the supersymmetry parameter and other super-covariant field strengths. This non-invariance under supersymmetry can be canceled

[11]Since the equations of motion in the first order theory will be the same as that of the theory without gravitational Chern-Simons term, we will still call this theory as the cosmological topological massless supergravity. The nontrivial massive excitation arises when we go to the second order formulation of the theory with gravitational Chern-Simons term and we will call this second order formulation as the cosmological topological massive supergravity.

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by adding other terms to the action constructed out of the super-covariant field strengths such that its supersymmetry variation exactly cancels the supersymmetry variation of the term that we initially added to the action. Therefore, these supersymmetric covariant field strengths will form the basic building blocks for constructing higher derivative terms invariant under the standard supersymmetry transformations (2.10), (2.11), (2.12). We will look at the effect of such terms on the boundary S-matrix coming from the standard supergravity action (which we shall compute in section 5) in section 4.

However, as discussed earlier there can be other higher derivative terms which can still render the full action to be invariant under a modified set of supersymmetry transformation laws. Such terms will not necessarily be constructed out of the super-covariant field strengths. We will also look at the effect of such terms on the standard boundary S-matrix towards the end of section 4

It is easy to see that G_{MN} and $\bar{G}_{MN}$ defined in (2.13) are already covariant with respect to the supersymmetry transformations (2.10). Hence, the fully covariantized Rarita-Schwinger field strength $\widetilde{G}_{MN}$ is the same as original Rarita-Schwinger field strength G_{MN}

$$\widetilde{G}_{MN} = G_{MN} \tag{3.1}$$

The gauge field strength $F_{MN} \equiv 2\partial_{[M}A_{N]}$ is not covariant with respect to the supersymmetry transformations (2.10). The covariantization of F_{MN} with respect to supersymmetry is obtained as

$$\widetilde{F}_{MN} \equiv 2\partial_{[M}A_{N]} - 2\delta^{(Q)}_{\frac{1}{2}\psi_{[M}}A_{N]} \tag{3.2}$$

Using the supersymmetry transformation (2.10), one finds that $\widetilde{F}_{MN}$ is same as $\widehat{F}_{MN}$ defined in (2.13), i.e

$$\widetilde{F}_{MN} = \widehat{F}_{MN} = F_{MN} - \frac{1}{2}\bar{\psi}_{[M}\psi_{N]} \tag{3.3}$$

Now we need to covariantize the Riemann tensor. The Riemann tensor $R_{MN}{}^{ab}$ (which is the field strength associated with ω_M^{ab}) defined in (2.2) is not covariant with respect to the supersymmetry transformations (2.10) and we need to covariantize it. This is obtained as

$$\widetilde{\mathcal{R}}_{MN}{}^{ab} = R_{MN}{}^{ab} - 2\delta^{(Q)}_{\frac{1}{2}\psi_{[M}}\omega_{N]}{}^{ab}, \tag{3.4}$$

Using the supersymmetry transformations (2.11) we find

$$\begin{aligned}\widetilde{\mathcal{R}}_{MN}{}^{ab} &= \mathcal{R}_{MN}{}^{ab} + \frac{i}{16m}\left[\bar{\psi}_{[M}\gamma_{N]}G^{ab} - \bar{G}^{ab}\gamma_{[N}\psi_{M]}\right] \\ &\quad + \frac{i}{8m}\left[\bar{\psi}_{[M}\gamma^{[a}G_{N]}{}^{b]} - \bar{G}_{[N}{}^{[b}\gamma^{a]}\psi_{M]}\right]\end{aligned} \tag{3.5}$$

The prescriptions (3.2) and (3.4) for covariantization of the field strengths have been worked out by looking at the supersymmetry transformations considered in section 2. One can easily check that the field strengths reproduced by such prescription are indeed covariant with respect to the standard supersymmetry transformations (2.10), (2.11), (2.12).

4. Effect of higher derivative terms

In this section we will look at the effect of higher derivative terms in the action on the boundary S-matrix computed from the standard supergravity action.[12] The most crucial thing to note is that by AdS/CFT correspondence [18, 19], the action evaluated for on-shell configuration of fields, with specified boundary conditions, is the partition function for evaluating correlators in the boundary, with the boundary values acting as sources for relevant correlators.[13] Thus if some terms vanish as a result of using equations of motion, it will not contribute to the boundary S-matrix.

First let us consider the higher derivative terms appearing in $\mathcal{S}_1$ which is needed for the supersymmetrization of the gravitational Chern-Simons term as argued in section 2. This term, as we saw in section 2, is constructed out of $\widehat{\mathcal{R}}_{MN}^{ab}$, $\widehat{F}_{MN}$ or G_{MN}. So the contribution of this term to the equations of motion will be terms containing $\widehat{\mathcal{R}}_{MN}^{ab}$, $\widehat{F}_{MN}$, G_{MN} and/or their super-covariant derivatives. Such terms will necessarily vanish when the original equation of motion (2.13)–(2.16) are satisfied. Therefore, the solutions obtained from $\mathcal{S}_0$ will continue to hold and $\mathcal{S}_1$ will vanish for such solutions and hence $\mathcal{S}_1$ will not affect the boundary correlators obtained in section 5.

There can be other higher derivative terms that can be added to the action that are supersymmetric on their own under the standard supersymmetry transformations (2.10), (2.11), (2.12). Such terms, as argued in section 3, should be constructed out of the supersymmetric covariant field strengths $\widetilde{\mathcal{R}}_{MN}^{ab}$, $\widehat{F}_{MN}$, G_{MN} and/or their supercovariant derivatives.

Our claim is that these higher derivative terms will change the original equations of motion (2.13) by terms which will vanish when the original equations of motion (2.13) are satisfied. Therefore, the solution obtained for the original equations of motion will still solve the equations of motion in the presence of these higher derivative terms. This, in particular means that $\widehat{F}_{MN} = 0$ and $G_{MN} = 0$ will still continue to hold, as a result of which the higher derivative terms constructed out of these field strengths will vanish and hence will not affect the boundary correlators calculated in section 5.

Let us analyze our claim in a little detail. The higher derivative terms that we could add to the gauge+fermionic sector will be of the form

$$\begin{aligned}\lambda_1 \int d^3x\ \widehat{F}_{MN} K_1^{MN}[\widehat{F}, G, \widetilde{\mathcal{R}}, \mathcal{D}^{(n)}(\widetilde{\mathcal{R}}, \widehat{F}, G)] \\ +\lambda_2 \int d^3x\ \widehat{F}_{MN} K_2^{MN}[\mathcal{R}, D^{(n)}\mathcal{R}] \\ +\lambda_3 \int d^3x\ \bar{G}^M \Gamma_{MN}[\widehat{F}, G, \widetilde{\mathcal{R}}, \mathcal{D}^{(n)}(\widetilde{\mathcal{R}}, \widehat{F}, G), \gamma] G^N\end{aligned} \tag{4.1}$$

Let us understand the notations used in the above equation. $(\mathcal{D}^{(n)}) D^{(n)}$ are the n^{th} (su-

[12] We shall compute this standard boundary S-matrix involving correlation functions of R-symmetry current $J(z)$ and supersymmetry current $G^{(+)}(z), G^{(-)}(z)$ in section 5.

[13] We will discuss this in great detail in section 5 when we actually evaluate the boundary correlators.

per)covariant derivatives. $\bar{G}^M$ and G^M are defined as

$$G^M = \frac{1}{2}\epsilon^{MNP}G_{NP} \quad \bar{G}^M = \frac{1}{2}\epsilon^{MNP}\bar{G}_{NP} \tag{4.2}$$

Γ_{MN} is a bilinear, constructed out of various supercovariant field strengths, their supercovariant derivatives and gamma matrices.

In the first integral, K_1^{MN} is a function of $(\widehat{F}, G, \widetilde{\mathcal{R}}, \mathcal{D}^{(n)}(\widetilde{\mathcal{R}}, \widehat{F}, G))$. Each term of K_1 should have a factor of $\widehat{F}$, G or their supercovariant derivatives. When there are no factors of $\widehat{F}$, G or their supercovariant derivatives involved, need special care and hence has been written as a separate term in which K_2 just depends on $(\mathcal{R}, D^{(n)}\mathcal{R})$.[14] Since $\widehat{F}_{MN}$ is antisymmetric in M and N, K_2^{MN} should also be antisymmetric in M and N. This implies that we cannot construct K_2^{MN} purely out of $\mathcal{R}_{MN}$ because of the symmetric nature of $\mathcal{R}_{MN}$. We have to involve covariant derivatives of $\mathcal{R}_{MN}$ in each term of K_2. Therefore, it is obvious that the contribution of the higher derivative terms to the equations of motion of A_M and ψ_M will have a factor of $\widehat{F}_{MN}$, G_{MN}, $D_M\mathcal{R}_{NP}$ and/or their supercovariant derivatives in it. Similarly it can also be argued that the contribution to the $\widetilde{\mathcal{R}}$ equation of motion from the higher derivatives terms should have the above factors. Just simply factors of $\mathcal{R}_{NP}$ will not contribute to any equation of motion. It has to be accompanied by at least one of the above factors.[15] When the original equations of motion (2.13)–(2.16) are satisfied, all these additional contributions to the equations of motion vanish. This justifies our earlier claim that the solutions obtained for the original equations of motion will still solve the equations of motion in the presence of these higher derivative terms and that the correlators calculated in section 5 in the gauge+fermionic sector will not be affected by these terms. We now use the same supersymmetry argument of [14] to argue for the non-renormalization of the stress tensor correlators. The crux of the argument is that the stress tensor correlators are related to the current correlators by supersymmetry. And since by our argument the current correlators do not get renormalized, this implies that the stress tensor correlators also do not get renormalized.

All the above discussions on the non-renormalization of the boundary S-matrix was done in the presence of higher derivative terms which rendered the full action to be invariant under the original supersymmetry transformation laws. However, as we have said in section 1 and 3, there can be other higher derivative terms which can be added that can keep the full action invariant under a modified set of supersymmetry transformation laws. As argued in [15] such terms can be removed by field redefinition and in terms of the field redefined variables, the action is invariant under the original supersymmetry

[14]For supersymmetric invariance K_2 should be constructed out of the supercovariant Riemann tensor $\widetilde{\mathcal{R}}$ and its supercovariant derivatives $\mathcal{D}^{(n)}\widetilde{\mathcal{R}}$ but the difference between the supercovariant Riemann tensor $\widetilde{\mathcal{R}}$, $\mathcal{D}^{(n)}\widetilde{\mathcal{R}}$ and $\mathcal{R}$, $D^{(n)}\mathcal{R}$ are terms proportional to G_{MN} and we have included such terms already in K_1^{MN}.

[15]The crucial observation in this analysis was that we cannot construct an antisymmetric K_2^{MN} purely out of $\mathcal{R}_{NP}$ because $\mathcal{R}_{NP}$ is symmetric. We have to involve covariant derivatives of $\mathcal{R}_{NP}$. The situation would have been drastically different if we were in more than three dimension. In that case we could construct an antisymmetric second rank tensor by using Riemann and Ricci tensors. But in three dimension Riemann tensor is not independent and is determined in terms of Ricci tensor and Ricci scalar.

transformation laws. Therefore, such terms will not necessarily be constructed out of the super-covariant field strengths.

Before proceeding further, first let us try to understand what sort of terms can be removed by redefining A and ψ. From (2.27) we get

$$\begin{aligned} \frac{\delta \mathcal{S}_0}{\delta A_M} &= -a_L \epsilon^{MNP} \widehat{F}_{NP} \\ \frac{\delta \mathcal{S}_0}{\delta \psi_M} &= \frac{i a_L}{2} \epsilon^{MNP} \bar{G}_{NP} = i a_L \bar{G}^M \\ \frac{\delta \mathcal{S}_0}{\delta \bar{\psi}_M} &= \frac{i a_L}{2} \epsilon^{MNP} G_{NP} = i a_L G^M \end{aligned} \tag{4.3}$$

Therefore the terms that can be removed by redefining A and ψ are of the form

$$\lambda_1 \int d^3x \ \widehat{F}_{MN} K^{MN} + \lambda_2 \int d^3x \ \bar{G}^M \Gamma_{MN} G^N \tag{4.4}$$

But contrary to (4.1), K^{MN} and Γ_{MN} appearing above are not necessarily constructed out of the super-covariant field strengths and their super-covariant derivatives. Since such terms can be removed by field redefinition, the full action can be invariant in their presence under a modified set of supersymmetry transformation laws. General coordinate invariance and U(1) invariance however suggests that K^{MN} and Γ_{MN} should be constructed out of F_{MN}, $\bar{\psi}_M \psi_N$, $\bar{\psi}\gamma\psi$, $\bar{G}_M G_N$, $\bar{G}\gamma G$ R_{MN} and their covariant derivatives. In special cases, K^{MN} and Γ_{MN} can be functions of the super-covariant field strengths and their super-covariant derivatives and (4.4) coincides with (4.1). Such terms as we have seen will not modify the equations of motion and will vanish on-shell and hence not modify the boundary S-matrix. But a general higher derivative term (4.4) can, in principle modify the equations of motion, and will not vanish on-shell, and hence change the boundary S-matrix. But since such terms can be removed by field redefinition, the boundary S-matrix can be related to the boundary S-matrix calculated from the standard supergravity action by a unitary transformation and such a unitary transformation is exactly identity for special class of higher derivative terms considered in (4.1)

5. Boundary S-matrix

So long we have been discussing the effects of higher derivative terms on the standard boundary S-matrix. Now for the sake of completeness, we shall evaluate this standard boundary S-matrix involving two and three point correlators of the weight "1" R-symmetry current ($J(z)$) and the two weight "$\frac{3}{2}$" supersymmetry currents ($G^{(+)}(z)$ and $G^{(-)}(z)$) in the dual conformal field theory. Here the superscript "(+)" and "(−)" denote the charges of the corresponding operators with respect to the global U(1), which is a symmetry of the theory.

There has been earlier works [29 − 31] which deals with Rarita-Schwinger fields in a general AdS_{d+1}/CFT_d correspondence. All the above works considered free massless as well as massive Rarita-Schwinger fields without coupling to any other fields in the bulk apart from gravity. But in a theory of extended supergravity, the Rarita Schwinger field couples to gravity as well as gauge field in the bulk. Obtaining the solution to the coupled field equations in a general dimension will be a monstrous task. However, we will see that in 3 dimensions the field equations can be written in a form notation and hence solving the coupled equation becomes somewhat simple and the coupling to gauge field can be taken care of, in a order by order fashion.

We begin this section by writing the supergravity action in Euclidean space

$$\begin{aligned}\mathcal{S} = ia_L &\int d^3x\ \epsilon^{MNP}\left[\frac{1}{2}B_M{}^a\partial_N B_P{}^b\delta_{ab} + \frac{i}{6}\varepsilon_{abc}B_M{}^aB_N{}^bB_P{}^c\right]\\ -ia_R &\int d^3x\ \epsilon^{MNP}\left[\frac{1}{2}B'_M{}^a\partial_N B'_P{}^b\delta_{ab} + \frac{i}{6}\varepsilon_{abc}B'_M{}^aB'_N{}^bB'_P{}^c\right]\\ +i\frac{a_L}{2} &\int d^3x\ \epsilon^{MNP}A_M\partial_N A_P\\ +\frac{a_L}{4} &\int d^3x\ \epsilon^{MNP}\left(\bar\psi_M\left(\mathcal{D}_N\psi_P\right) - \left(\mathcal{D}_N\bar\psi_M\right)\psi_P\right)\end{aligned} \tag{5.1}$$

Where[16]

$$\begin{aligned}B_M{}^a &= \frac{i}{2}\varepsilon^{abc}\omega_{Mbc} - me_M{}^a,\\ B'_M{}^a &= \frac{i}{2}\varepsilon^{abc}\omega_{Mbc} + me_M{}^a,\\ \mathcal{D}_M\psi_N &= \partial_M\psi_N - \frac{1}{2}B_M{}^a\gamma_a\psi_N + \frac{i}{2}A_M\psi_N,\\ \mathcal{D}_M\bar\psi_N &= \partial_M\bar\psi_N + \frac{1}{2}B_M{}^a\bar\psi_N\gamma_a - \frac{i}{2}A_M\bar\psi_N,\end{aligned} \tag{5.2}$$

The gamma matrices satisfies the algebra:

$$\begin{aligned}\{\gamma_a,\gamma_b\} &= 2\delta_{ab},\\ [\gamma_a,\gamma_b] &= -2i\varepsilon_{abc}\gamma_c,\end{aligned} \tag{5.3}$$

We will not be interested in obtaining any correlators involving the stress tensor. Therefore gravity just enters as a global AdS_3 background. The metric of Euclidean AdS_3 written in Poincare patch coordinate system is

$$\begin{aligned}ds^2 &= \frac{1}{m^2(x^0)^2}((dx^0)^2 + (dx^1)^2 + (dx^2)^2)\\ &= \frac{1}{m^2(x^0)^2}((dx^0)^2 + dzd\bar z), \qquad z = x^1 + ix^2\end{aligned} \tag{5.4}$$

The gauge field and Rarita Schwinger equations are the same as obtained in the Lorentzian case (2.13). The equations of motion can be written in a form notation

$$\begin{aligned}dA &= \frac{1}{4}\bar\psi\wedge\psi,\\ d\psi - \frac{1}{2}B^a\gamma_a\wedge\psi &= -\frac{i}{2}A\wedge\psi,\\ d\bar\psi + \frac{1}{2}B^a\wedge\bar\psi\gamma_a &= \frac{i}{2}A\wedge\bar\psi\end{aligned} \tag{5.5}$$

[16]Note the change in the definition of $B_M{}^a$ and $B'_M{}^a$ compared to the lorentzian case (2.6), (2.8).

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Let us now outline the important steps involved in the calculation of the correlation function

1. Solve the equation (5.5) order by order i.e. first solve the leading order equations, where we set the r.h.s. appearing in all the equations to zero. Then obtain the first order corrections by putting the solution obtained for leading order in the r.h.s. and so on. For our purpose we will obtain just the first order correction. We will need to impose the following gauge conditions on ψ and $\bar{\psi}$ to obtain the solution

$$\gamma^M \psi_M = 0 \qquad \bar{\psi}_M \gamma^M = 0 \tag{5.6}$$

2. From the boundary behavior ($x^0 \to 0$), we need to figure out boundary values of which fields has the appropriate conformal dimension, so as to source the corresponding operators in the boundary and then impose boundary conditions on those fields.

3. Obtain the solutions obtained in (1) in terms of the boundary conditions imposed in (2) but upto terms quadratic in the boundary values.

4. We need to add boundary term to the action (5.1) for consistency requirements. The reason is the following. We have imposed boundary conditions on some components of the fields and left others to vary. The variation of the action (5.1) will have a boundary piece involving the variation of those components of the fields on which we have not imposed any boundary conditions and hence will not vanish. Therefore we need to add a boundary piece to the action such that its variation exactly cancels this contribution.

5. Evaluate the bulk action (5.1) as well as boundary action obtained in (4) as a functional of the boundary values but just upto terms cubic in the boundary values.

6. Obtain the correlation function in the boundary using the prescription given in [18, 19]

Before proceeding with the rest of the section, there is a subtle issue which we would like to address. ${B_M}^a$ and ${B'_M}^a$ defined in (5.2) has ${\omega_M}^a$ in it which satisfies the constraint equation (2.3). Because of the presence of $\bar{\psi}_M \gamma^a \psi_N$ in the right hand side of the equation (2.3), ${\omega_M}^a$ and hence ${B_M}^a$ and ${B'_M}^a$ do not have purely gravitational contribution. But the gravitational contribution and fermionic contribution to ${\omega_M}^a$ can be decoupled as ${\omega_M}^a = \omega_M^{(e)a} + \omega_M^{(f)a}$.
$\omega_M^{(e)a}$ satisfies the standard torsionless constraint equation and $\omega_M^{(f)a}$ contributes to the torsion part of the equation (2.3)

$$\begin{aligned} de^a + \omega^{(e)ab} \wedge e_b &= 0, \\ \omega^{(f)ab} \wedge e_b &= -\frac{i}{8m} \bar{\psi} \wedge \gamma^a \psi \end{aligned} \tag{5.7}$$

Clearly $\omega_M^{(f)a}$ will be $\mathcal{O}(\bar{\psi}\gamma\psi)$ and its contribution to the equation of motion (5.5) through ${B_M}^a$ and ${B'_M}^a$ will be cubic in ψ. Since we would like to obtain the solution upto terms quadratic in the fields, we will neglect this term and use the standard torsionless spin connection in ${B_M}^a$ and ${B'_M}^a$ for solving the equations of motion (5.5). Since $\omega_M^{(f)}$ has a quadratic dependence on the fermions, *a Priori* it seems that, in the evaluation of the action for the on-shell configuration of fields as prescribed in step 5, as if $\omega_M^{(f)}$ will give a contribution to the on-shell action quadratic in the boundary values of the fermions through the dependence of the action (5.1) on ${B_M}^a$ and ${B'_M}^a$ as

$$\begin{aligned} & ia_L \int d^3x\, \epsilon^{MNP}\, \omega_M^{(f)a} \left[\partial_N B^{(e)}_{Pa} + \frac{i}{2}\varepsilon_{abc} B_N^{(e)b} B_P^{(e)c}\right] \\ & \qquad -ia_R \int d^3x\, \epsilon^{MNP}\, \omega_M^{(f)a} \left[\partial_N B^{(e)\prime}_{Pa}\delta_{ab} + \frac{i}{2}\varepsilon_{abc} B_N^{(e)\prime b} B_P^{(e)\prime c}\right] \end{aligned} \tag{5.8}$$

Where

$$\begin{aligned} B_N^{(e)b} &= \omega_N^{(e)b} - m{e_N}^b, \\ B_N^{(e)\prime b} &= \omega_N^{(e)b} + m{e_N}^b \end{aligned} \tag{5.9}$$

But one can check that for AdS_3 background the terms inside the square bracket of the above integral (5.8) vanishes and hence will not give a quadratic contribution to the evaluation of the on-shell action prescribed in step 5. However, one has to be careful while obtaining quartic and higher contribution. One has to take the effect of the torsion in the spin connection into account. But since we are only interested in obtaining the on-shell action upto terms cubic in the fields, we will use the torsionless spin connection while solving the equation of motion (5.5) and while evaluating the action for the on-shell configuration of fields we will neglect contribution from the first two terms in the action (5.1).

The stepwise analysis of step 1 to step 5 has been given in appendix A. But we will just outline the important results here and obtain the correlators. From step 2 we see that we need to impose the following boundary conditions [17]

$$\begin{aligned} \lim_{x^0\to 0} A_{\bar z} &= A^{(0)}_{\bar z}(\vec z), \\ \lim_{x^0\to 0} (x^0)^{\frac12}\psi^{(1)}_{\bar z} &= \Theta^{(-)}_{\bar z}(\vec z), \\ \lim_{x^0\to 0} (x^0)^{\frac12}\bar\psi^{(2)}_{\bar z} &= \Theta^{(+)}_{\bar z}(\vec z) \end{aligned} \tag{5.10}$$

From step 4 we get the following boundary action

$$\begin{aligned} \mathcal{S}_{\texttt{bndy}} &= \mathcal{S}_{\texttt{bndy}}[\psi,\bar\psi] + \mathcal{S}_{\texttt{bndy}}[A] \\ \mathcal{S}_{\texttt{bndy}}[\psi,\bar\psi] &= -\frac{ia_L}{2}\int d^2\vec z\ \left(\bar\psi^{(1)}_z(x^0,\vec z)\psi^{(1)}_{\bar z}(x^0,\vec z) + \bar\psi^{(2)}_{\bar z}(x^0,\vec z)\psi^{(2)}_z(x^0,\vec z)\right)\Big|_{x^0=0} \\ \mathcal{S}_{\texttt{bndy}}[A] &= a_L\int d^2\vec z\ A_z(\vec z,x^0)A_{\bar z}(\vec z,x^0)|_{x^0=0} \end{aligned} \tag{5.11}$$

Here the superscripts (1) and (2) in the Rarita-Schwinger fields represents the spinor components. The measure $d^2\vec z$ is defined as

$$d^2\vec z = dx^1 dx^2 \tag{5.12}$$

[17] The superscripts in the corresponding boundary values represents charge with respect to the U(1) symmetry in the theory.

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After obtaining the solution to the equations of motion in step 3 subject to the boundary conditions (5.10) upto terms quadratic in the boundary values, we proceed to step 5 where we evaluate the action (5.1) along with the boundary action (5.11) for these on-shell configuration of fields. We neglect the contribution coming from the first two terms in (5.1) as we have argued before. We get

$$\begin{aligned}\mathcal{S}[A^{(0)}_{\bar{z}},\Theta^{(+)}_{\bar{z}},\Theta^{(-)}_{\bar{z}}] = &-\frac{a_L}{\pi}\int d^2\ \vec{z}\ d^2\vec{w}\ \frac{1}{(z-w)^2}\ A^{(0)}_{\bar{z}}(\vec{w})\ A^{(0)}_{\bar{z}}(\vec{z}) \\ &-\frac{2ia_L}{\pi}\int d^2\vec{z}\ d^2\vec{w}\ \frac{1}{(z-w)^3}\ \Theta^{(+)}_{\bar{z}}(\vec{w})\ \Theta^{(-)}_{\bar{z}}(\vec{z}) \\ &-\frac{a_L}{\pi^2}\int d^2\vec{z}\ d^2\vec{w}\ d^2\vec{v}\ \frac{1}{(z-w)(z-v)(w-v)^2}\ A^{(0)}_{\bar{z}}(\vec{z})\ \Theta^{(+)}_{\bar{z}}(\vec{v})\ \Theta^{(-)}_{\bar{z}}(\vec{w})\end{aligned} \tag{5.13}$$

Then according to AdS/CFT conjecture [18, 19], we have

$$\exp(-\mathcal{S}(A,\psi,\bar{\psi})) = \left\langle \exp\left(\frac{1}{2\pi}\int_{\partial} J(\vec{z})A^{(0)}_{\bar{z}}(\vec{z}) + G^{(+)}(\vec{z})\Theta^{(-)}_{\bar{z}}(\vec{z}) + G^{(-)}(\vec{z})\Theta^{(+)}_{\bar{z}}(\vec{z})\right)\right\rangle \tag{5.14}$$

This implies the following two and three point correlation functions

$$\begin{aligned}\langle J(\vec{z_1})J(\vec{z_2})\rangle &= (2\pi)^2\frac{\delta}{\delta A^{(0)}_{\bar{z}}(\vec{z_1})}\frac{\delta}{\delta A^{(0)}_{\bar{z}}(\vec{z_2})}e^{-\mathcal{S}}\Bigg|_{(A^{(0)}_{\bar{z}}(\vec{z}),\Theta^{(+)}_{\bar{z}}(\vec{z}),\Theta^{(-)}_{\bar{z}}(\vec{z}))=0} \\ \left\langle G^{(+)}(\vec{z_1})G^{(-)}(\vec{z_2})\right\rangle &= (2\pi)^2\frac{\delta}{\delta\Theta^{(-)}_{\bar{z}}(\vec{z_1})}\frac{\delta}{\delta\Theta^{(+)}_{\bar{z}}(\vec{z_2})}e^{-\mathcal{S}}\Bigg|_{(A^{(0)}_{\bar{z}}(\vec{z}),\Theta^{(+)}_{\bar{z}}(\vec{z}),\Theta^{(-)}_{\bar{z}}(\vec{z}))=0} \\ \left\langle J(\vec{z_1})G^{(+)}(\vec{z_2})G^{(-)}(\vec{z_3})\right\rangle &= (2\pi)^3\frac{\delta}{\delta A^{(0)}_{\bar{z}}(\vec{z_1})}\frac{\delta}{\delta\Theta^{(-)}_{\bar{z}}(\vec{z_2})}\frac{\delta}{\delta\Theta^{(+)}_{\bar{z}}(\vec{z_3})}e^{-\mathcal{S}}\Bigg|_{(A^{(0)}_{\bar{z}}(\vec{z}),\Theta^{(+)}_{\bar{z}}(\vec{z}),\Theta^{(-)}_{\bar{z}}(\vec{z}))=0}\end{aligned} \tag{5.15}$$

Using (5.13) in the above equation (5.15), we get

$$\begin{aligned}\langle J(\vec{z_1})J(\vec{z_2})\rangle &= 8a_L\pi\frac{1}{(z_1-z_2)^2} \\ \left\langle G^{(+)}(\vec{z_1})G^{(-)}(\vec{z_2})\right\rangle &= 8ia_L\pi\frac{1}{(z_1-z_2)^3} \\ \left\langle J(\vec{z_1})G^{(+)}(\vec{z_2})G^{(-)}(\vec{z_3})\right\rangle &= 8a_L\pi\frac{1}{(z_1-z_2)(z_1-z_3)(z_2-z_3)^2}\end{aligned} \tag{5.16}$$

These are the expected conformal field theory results.

6. Discussion

We looked at the $\mathcal{N} = (2,0)$ supergravity action in the presence of gravitational Chern-Simons term in the first order and second order formulation. We found some inherent differences between the two formulations. In particular, supersymmetry forced us to add a particular type of higher derivative term in the second order formulaton and the equations of motion in the second order formulation get modified and retain as a subset, the solutions to the equations of motion in the first order formulation.

We then constructed various field strengths covariant with respect to the supersymmetry transformations under which the standard supergravity action is invariant. These field strengths, as we argued, will form the basic building blocks for constructing higher derivative terms which will be invariant under the supersymmetry transformations which rendered the standard supergravity action to be invariant.

We then looked at the effect of higher derivative terms on the boundary S-matrix which shall arise from the standard supergravity action. We saw that there are a special class of higher derivative terms constructed out of the super-covariant field strengths constructed in section 3. Such terms will not modify the equations of motion and will hence vanish on-shell and not modify the correlation functions. However, there can be other higher derivative terms, in the presence of which, the full action will be invariant under a modified set of supersymmetry transformation laws. Such terms can, in principle change the equations of motion and will not vanish on-shell as a consequence, and hence change the correlation functions. But since such terms can be removed by field redefinition, and in terms of the redefined field variables the correlation functions are the same as that obtained for the standard action, this implies that the correlation functions obtained in the presence of such higher derivative terms will be related to the standard correlation functions by a unitary transformation. Such a unitary transformation is exactly identity when the higher derivative terms are constructed out of the super-covariant field strengths and their super-covariant derivatives.

In the end, for the sake of completeness, we computed the standard boundary S-matrix describing correlation functions of weight "1" U(1) current and weight "$\frac{3}{2}$" supersymmetry current and we found the expected conformal field theory results for the two and three point functions.

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A. Boundary S-matrix analysis

Here we will give stepwise analysis of the steps outlined in section 5, for obtaining the boundary S-matrix in the gauge+fermionic sector.

1. We will present the solution obtained for the equation (5.5) till first order iteration. For this we need to calculate the vierbeins and the torsionless spin connection from

the AdS_3 metric (5.4) and put them in the expression for ${B_M}^a$. We get

$$\begin{aligned} B^{\hat{0}} &= -\frac{1}{x^0}dx^0, \\ B^{\hat{1}} &= -\frac{1}{x^0}dx^1 - \frac{i}{x^0}dx^2 = -\frac{1}{x^0}dz, \\ B^{\hat{2}} &= \frac{i}{x^0}dx^1 - \frac{1}{x^0}dx^2 = \frac{i}{x^0}dz \end{aligned} \tag{A.1}$$

After putting (A.1) in the equations of motion (5.5), the leading order equations take the form[18]

$$\begin{aligned} dA^0 &= 0, \\ d\psi^0 + \frac{1}{2x^0}dx^0 \wedge \gamma_{\hat{0}}\psi^0 + \frac{1}{x^0}dz \wedge \gamma_{\hat{z}}\psi^0 &= 0, \\ d\bar{\psi}^0 - \frac{1}{2x^0}dx^0 \wedge \bar{\psi}^0\gamma_{\hat{0}} - \frac{1}{x^0}dz \wedge \bar{\psi}^0\gamma_{\hat{z}} &= 0 \end{aligned} \tag{A.2}$$

The solution to the first equation is simple and the solution to the last two equations can be obtained after a little algebraic manipulation. We get

$$\begin{aligned} A^0 &= d\rho, \\ \psi^{0(1)} &= (x^0)^{-\frac{1}{2}}\left(d\eta + \phi dz\right), \\ \psi^{0(2)} &= (x^0)^{\frac{1}{2}}d\phi, \\ \bar{\psi}^{0(2)} &= (x^0)^{-\frac{1}{2}}\left(d\bar{\eta} - \bar{\phi}dz\right), \\ \bar{\psi}^{0(1)} &= (x^0)^{\frac{1}{2}}d\bar{\phi}, \end{aligned} \tag{A.3}$$

Where, as before the spinor index has been kept in the superscript and in () braces and the 0 to the left of it just denotes the leading order piece. Here ρ, η, ϕ, $\bar{\eta}$, $\bar{\phi}$ are some arbitrary functions of $(x^0,\ z,\ \bar{z})$. ρ is an ordinary function whereas η, ϕ, $\bar{\eta}$, $\bar{\phi}$ are grassman functions.

Now we obtain the first order correction $A^1, \psi^1, \bar{\psi}^1$ to the above equation as

$$\begin{aligned} dA^1 &= \frac{1}{4}\bar{\psi}^0 \wedge \psi^0, \\ d\psi^1 + \frac{1}{2x^0}dx^0 \wedge \gamma_{\hat{0}}\psi^1 + \frac{1}{x^0}dz \wedge \gamma_{\hat{z}}\psi^1 &= -\frac{i}{2}A^0 \wedge \psi^0, \\ d\bar{\psi}^1 - \frac{1}{2x^0}dx^0 \wedge \bar{\psi}^1\gamma_{\hat{0}} - \frac{1}{x^0}dz \wedge \bar{\psi}^1\gamma_{\hat{z}} &= \frac{i}{2}A^0 \wedge \bar{\psi}^0, \end{aligned} \tag{A.4}$$

We get

$$\begin{aligned} A^1 &= \frac{1}{4}\bar{\phi}d\eta + \frac{1}{4}\bar{\eta}d\phi + \frac{1}{4}\bar{\phi}\phi dz, \\ \psi^{1(1)} &= -\frac{i}{2}(x^0)^{-\frac{1}{2}}\rho(d\eta + \phi dz), \\ \psi^{1(2)} &= -\frac{i}{2}(x^0)^{\frac{1}{2}}\rho d\phi, \end{aligned}$$

[18]We use $\gamma_{\hat{0}} = \sigma_3$, $\gamma_{\hat{1}} = \sigma_1$, $\gamma_{\hat{2}} = -\sigma_2$, $\gamma_{\hat{z}} \equiv \frac{1}{2}\left(\gamma_{\hat{1}} - i\gamma_{\hat{2}}\right)$, $\gamma_{\hat{\bar{z}}} \equiv \frac{1}{2}\left(\gamma_{\hat{1}} + i\gamma_{\hat{2}}\right)$

$$\bar{\psi}^{1(2)} = \frac{i}{2}(x^0)^{-\frac{1}{2}}\rho(d\bar{\eta} - \bar{\phi}dz),$$
$$\bar{\psi}^{1(1)} = \frac{i}{2}(x^0)^{\frac{1}{2}}\rho d\bar{\phi}, \tag{A.5}$$

Thus the full solution till first order iteration is

$$\begin{aligned}
A &= d\rho + \frac{1}{4}\bar{\phi}d\eta + \frac{1}{4}\bar{\eta}d\phi + \frac{1}{4}\bar{\phi}\phi dz, \\
\psi^{(1)} &= (x^0)^{-\frac{1}{2}}\left(1 - \frac{i}{2}\rho\right)(d\eta + \phi dz), \\
\psi^{(2)} &= (x^0)^{\frac{1}{2}}\left(1 - \frac{i}{2}\rho\right)d\phi, \\
\bar{\psi}^{(1)} &= (x^0)^{\frac{1}{2}}\left(1 + \frac{i}{2}\rho\right)d\bar{\phi}, \\
\bar{\psi}^{(2)} &= (x^0)^{-\frac{1}{2}}\left(1 + \frac{i}{2}\rho\right)(d\bar{\eta} - \bar{\phi}dz)
\end{aligned} \tag{A.6}$$

After imposing the gauge condition (5.6), we get the following equations for ϕ, η, $\bar{\phi}$ and $\bar{\eta}$

$$\begin{aligned}
&\partial_0\eta + 2x^0\partial_{\bar{z}}\phi = 0 \\
&x^0\partial_0\phi - 2\left(\partial_z\eta + \phi\right) = 0 \\
&\partial_0\bar{\eta} - 2x^0\partial_{\bar{z}}\bar{\phi} = 0 \\
&x^0\partial_0\bar{\phi} + 2\left(\partial_z\bar{\eta} - \bar{\phi}\right) = 0
\end{aligned} \tag{A.7}$$

The above equations can be solved after decoupling them to obtain a second order equation and then using separation of variables to separate the x^0 equation and $(z, \bar{z})$ equation. The result is

$$\begin{aligned}
\eta &= \frac{1}{2\pi}\int d^2\vec{p}(x^0p)^2K_2(x^0p)e^{i\vec{p}.\vec{z}}\mathcal{A}_\eta(\vec{p}), \\
\phi &= \frac{1}{2\pi}\int d^2\vec{p}(x^0p)K_1(x^0p)e^{i\vec{p}.\vec{z}}(-2ip_z)\mathcal{A}_\eta(\vec{p}), \\
\bar{\eta} &= \frac{1}{2\pi}\int d^2\vec{p}(x^0p)^2K_2(x^0p)e^{i\vec{p}.\vec{z}}\mathcal{A}_{\bar{\eta}}(\vec{p}), \\
\bar{\phi} &= \frac{1}{2\pi}\int d^2\vec{p}(x^0p)K_1(x^0p)e^{i\vec{p}.\vec{z}}(2ip_z)\mathcal{A}_{\bar{\eta}}(\vec{p})
\end{aligned} \tag{A.8}$$

Where

$$\vec{p} \equiv (p_z, p_{\bar{z}}) \quad p^2 \equiv |\vec{p}|^2 \equiv 4p_zp_{\bar{z}} \quad \vec{p}.\vec{z} \equiv p_zz + p_{\bar{z}}\bar{z} \tag{A.9}$$

The measure $d^2\vec{p}$ is defined as

$$d^2\vec{p} \equiv dp_1dp_2 \tag{A.10}$$

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Where p_1 and p_2 are related to p_z and $p_{\bar{z}}$ as

$$
\begin{aligned}
p_1 &\equiv \frac{1}{2}\mathtt{Re}(p_z), \\
p_2 &\equiv -\frac{1}{2}\mathtt{Im}(p_z) \\
\texttt{i.e.} \qquad p_z &= \frac{1}{2}(p_1 - ip_2), \\
p_{\bar{z}} &= \frac{1}{2}(p_1 + ip_2), \\
\texttt{Hence} \qquad p^2 &\equiv 4p_z p_{\bar{z}} = p_1^2 + p_2^2 \\
\texttt{And} \qquad d^2\vec{p} \equiv dp_1 dp_2 &= \begin{vmatrix} \frac{\partial p_1}{\partial p_z} & \frac{\partial p_1}{\partial p_{\bar{z}}} \\ \frac{\partial p_2}{\partial p_z} & \frac{\partial p_2}{\partial p_{\bar{z}}} \end{vmatrix} dp_z dp_{\bar{z}} = -2i dp_z dp_{\bar{z}}
\end{aligned} \tag{A.11}
$$

Here K_1 and K_2 are the "modified Bessel's functions" of order 1 and 2 respectively. They behave asymptotically ($x \to 0$) as

$$
\begin{aligned}
K_2(x) &= \frac{2}{x^2}\left[1 + \mathcal{O}(x^2)\right], \\
K_1(x) &= \frac{1}{x}\left[1 + \mathcal{O}(x^2)\right]
\end{aligned} \tag{A.12}
$$

It can be easily checked that the solutions (A.8) indeed satisfies the equations (A.7). For that, we need to use the following property of modified Bessel's functions

$$
\begin{aligned}
K_2'(x^0 p) &= -K_1(x^0 p) - \frac{2}{x^0 p}K_2(x^0 p) \\
K_1'(x^0 p) &= -K_2(x^0 p) + \frac{1}{x^0 p}K_1(x^0 p)
\end{aligned} \tag{A.13}
$$

2. We will now figure out the fields on which we should impose boundary conditions. Since we are working in $(2,0)$ theory, we should impose boundary conditions on the $\bar{z}$ components of the fields. The boundary conditions on the gauge field is unambiguous and is simply

$$
\lim_{x^0 \to 0} A_{\bar{z}}(x^0, \vec{z}) = A_{\bar{z}}^{(0)}(\vec{z}) \tag{A.14}
$$

Before trying to figure out what boundary conditions we should impose on the Rarita-Schwinger fields, let us outline an important result of [18]. According to [18], if a p-form field C behave as $(x^0)^{-\lambda}C_0$ near the boundary then the operator $\mathcal{O}$ that couples to C_0 in the boundary will have conformal dimensions $\Delta = d - p + \lambda$. Here d is the dimensions of the boundary. For our case $d = 2$ and $p = 1$ and we want sources for weight $\frac{3}{2}$ supersymmetry currents. This suggests $\lambda = \frac{1}{2}$. This behavior near the boundary is there for the fields $\psi^{(1)}$ and $\bar{\psi}^{(2)}$ as can be seen from (A.6). Thus we impose the following boundary conditions on the Rarita-Schwinger fields

$$
\begin{aligned}
\lim_{x^0 \to 0} (x^0)^{\frac{1}{2}} \psi_{\bar{z}}^{(1)} &= \Theta_{\bar{z}}^{(-)}(\vec{z}), \\
\lim_{x^0 \to 0} (x^0)^{\frac{1}{2}} \bar{\psi}_{\bar{z}}^{(2)} &= \Theta_{\bar{z}}^{(+)}(\vec{z})
\end{aligned} \tag{A.15}
$$

Following [14], we take ρ appearing in the solutions (A.6) to be

$$\rho = \int d^2\vec{w}\ K(\vec{z}, x^0, \vec{w})\mathcal{B}^{(0)}_{\bar{z}}(\vec{w}),$$
$$K(\vec{z}, x^0, \vec{w}) = \frac{1}{\pi}\left[\frac{\bar{z}-\bar{w}}{(x^0)^2 + |z-w|^2}\right] \tag{A.16}$$

The function $K(\vec{z}, x^0, \vec{w})$ satisfies the following properties on the boundary

$$\lim_{x^0\to 0} K = \frac{1}{\pi}\frac{1}{z-w},$$
$$\lim_{x^0\to 0} \partial_z K = -\frac{1}{\pi}\frac{1}{(z-w)^2},$$
$$\lim_{x^0\to 0} \partial_{\bar{z}} K = \delta^2(z-w), \tag{A.17}$$

3. We will use the above properties of K along with the behavior of the modified Bessel's functions K_1 and K_2 near the boundary to fix $\mathcal{B}^{(0)}_{\bar{z}}(\vec{w})$ appearing in (A.16) and $\mathcal{A}_\eta(\vec{p})$ and $\mathcal{A}_{\bar{\eta}}(\vec{p})$ appearing in (A.8) in terms of the boundary values so that the boundary conditions (A.14) and (A.15) are satisfied. We get

$$\mathcal{B}^{(0)}_{\bar{z}}(\vec{z}) = A^{(0)}_{\bar{z}}(\vec{z}) + \frac{1}{4\pi}\int d^2\vec{w}\ \frac{1}{(z-w)^2}\ \Theta^{(+)}_{\bar{z}}(\vec{w})\ \Theta^{(-)}_{\bar{z}}(\vec{z}) \tag{A.18}$$
$$\mathcal{A}_\eta(\vec{p}) = \frac{1}{2\pi}\int d^2\vec{w}\ \left(\frac{1}{2ip_{\bar{z}}}\right)\ e^{-i\vec{p}.\vec{w}}\ \Theta^{(-)}_{\bar{z}}(\vec{w})$$
$$+\frac{i}{4\pi^2}\int d^2\vec{w}\ d^2\vec{v}\ \left(\frac{1}{2ip_{\bar{z}}}\right)\ \left(\frac{1}{w-v}\right)\ e^{-i\vec{p}.\vec{w}}\ A^{(0)}_{\bar{z}}(\vec{v})\ \Theta^{(-)}_{\bar{z}}(\vec{w})$$
$$\mathcal{A}_{\bar{\eta}}(\vec{p}) = \frac{1}{2\pi}\int d^2\vec{w}\ \left(\frac{1}{2ip_{\bar{z}}}\right)\ e^{-i\vec{p}.\vec{w}}\ \Theta^{(+)}_{\bar{z}}(\vec{w})$$
$$-\frac{i}{4\pi^2}\int d^2\vec{w}\ d^2\vec{v}\ \left(\frac{1}{2ip_{\bar{z}}}\right)\ \left(\frac{1}{w-v}\right)\ e^{-i\vec{p}.\vec{w}}\ A^{(0)}_{\bar{z}}(\vec{v})\ \Theta^{(+)}_{\bar{z}}(\vec{w})$$

The above result, then fixes the solutions completely in terms of the boundary values. We get[19]

$$A_z = -\frac{1}{\pi}\int d^2\vec{w}\ \frac{1}{(z-w)^2}\ A^{(0)}_{\bar{z}}(\vec{w})$$
$$-\frac{1}{4\pi^2}\int d^2\vec{v}\ d^2\vec{w}\frac{1}{(z-w)^2(w-v)^2}\ \Theta^{(+)}_{\bar{z}}(\vec{v})\ \Theta^{(-)}_{\bar{z}}(\vec{w})$$
$$-\frac{1}{2\pi^2}\int d^2\vec{v}\ d^2\vec{w}\ \frac{1}{(z-w)^3(z-v)}\ \Theta^{(+)}_{\bar{z}}(\vec{v})\ \Theta^{(-)}_{\bar{z}}(\vec{w}) + \mathcal{O}\left((x^0)^2\right)$$
$$A_{\bar{z}} = A^{(0)}_{\bar{z}}(\vec{z}) + \mathcal{O}\left((x^0)^2\right)$$
$$(x^0)^{\frac{1}{2}}\psi^{(1)}_{\bar{z}} = \Theta^{(-)}_{\bar{z}}(\vec{z}) + \mathcal{O}\left((x^0)^2\right)$$
$$(x^0)^{\frac{1}{2}}\psi^{(1)}_{z} = \mathcal{O}\left((x^0)^2\right)$$

[19] In order to arrive at the results (A.19) we needed to do a lot of "p" integrals. One of such integral is outlined in the end of the section. The others can be similarly obtained

$$
\begin{aligned}
(x^0)^{-\frac{1}{2}}\psi_z^{(2)} &= -\frac{2}{\pi}\int d^2\vec{w}\ \frac{1}{(z-w)^3}\ \Theta_{\bar{z}}^{(-)}(\vec{w}) \\
&\quad -\frac{i}{\pi^2}\int d^2\vec{v}\ d^2\vec{w}\ \frac{1}{(z-w)^2(w-v)(z-v)}\ A_{\bar{z}}^{(0)}(\vec{v})\ \Theta_{\bar{z}}^{(-)}(\vec{w}) + \mathcal{O}\left((x^0)^2\right) \\
(x^0)^{-\frac{1}{2}}\psi_{\bar{z}}^{(2)} &= \mathcal{O}\left((x^0)^2\right) \\
(x^0)^{\frac{1}{2}}\bar{\psi}_{\bar{z}}^{(2)} &= \Theta_{\bar{z}}^{(+)}(\vec{z}) + \mathcal{O}\left((x^0)^2\right) \\
(x^0)^{\frac{1}{2}}\bar{\psi}_z^{(2)} &= \mathcal{O}\left((x^0)^2\right) \\
(x^0)^{-\frac{1}{2}}\bar{\psi}_z^{(1)} &= \frac{2}{\pi}\int d^2\vec{w}\ \frac{1}{(z-w)^3}\ \Theta_{\bar{z}}^{(+)}(\vec{w}) \\
&\quad -\frac{i}{\pi^2}\int d^2\vec{v}\ d^2\vec{w}\ \frac{1}{(z-w)^2(w-v)(z-v)}\ A_{\bar{z}}^{(0)}(\vec{v})\ \Theta_{\bar{z}}^{(+)}(\vec{w}) + \mathcal{O}\left((x^0)^2\right) \\
(x^0)^{-\frac{1}{2}}\bar{\psi}_{\bar{z}}^{(1)} &= \mathcal{O}\left((x^0)^2\right)
\end{aligned}
\tag{A.19}
$$

4. Now we will obtain the relevant boundary action that we need to add for consistency requirements mentioned in section 5. Varying the action (5.1) we get

$$
\begin{aligned}
\delta\mathcal{S} = [0]_{\texttt{on-shell}} &+ \frac{ia_L}{2}\lim_{\epsilon\to 0}\int_{\mathcal{M}_\epsilon} d^2\vec{z}\ \left(\delta\bar{\psi}_z^{(1)}(\epsilon,\vec{z})\psi_{\bar{z}}^{(1)}(\epsilon,\vec{z}) + \bar{\psi}_{\bar{z}}^{(2)}(\epsilon,\vec{z})\delta\psi_z^{(2)}(\epsilon,\vec{z})\right) \\
&- a_L\lim_{\epsilon\to 0}\int_{\mathcal{M}_\epsilon} d^2\vec{z}\ \delta A_z(\vec{z},\epsilon)A_{\bar{z}}(\vec{z},\epsilon)
\end{aligned}
\tag{A.20}
$$

Here $[0]_{\texttt{on-shell}}$ denotes the set of terms which vanish on-shell. In obtaining the above variation, we have used the fact that we have imposed boundary conditions (A.14), (A.15) and hence

$$
\lim_{\epsilon\to 0}\delta A_{\bar{z}}(\epsilon,\vec{z}) = \lim_{\epsilon\to 0}\delta\psi_{\bar{z}}^{(1)}(\epsilon,\vec{z}) = \lim_{\epsilon\to 0}\delta\bar{\psi}_{\bar{z}}^{(2)}(\epsilon,\vec{z}) = 0 \tag{A.21}
$$

Moreover from (A.19), we observe

$$
\lim_{\epsilon\to 0}\epsilon^{\frac{1}{2}}\psi_z^{(1)}(\epsilon,\vec{z}) = \lim_{\epsilon\to 0}\epsilon^{-\frac{1}{2}}\bar{\psi}_{\bar{z}}^{(1)}(\epsilon,\vec{z}) = \lim_{\epsilon\to 0}\epsilon^{\frac{1}{2}}\bar{\psi}_z^{(2)}(\epsilon,\vec{z}) = \lim_{\epsilon\to 0}\epsilon^{-\frac{1}{2}}\psi_{\bar{z}}^{(2)}(\epsilon,\vec{z}) = 0 \tag{A.22}
$$

The relations (A.22) are valid on-shell since these are obtained from (A.19) which are the solutions to the equations of motion. This along with (A.21) has been used in arriving at (A.20). We need to add a boundary action $S_{\texttt{bndy}}$ such that its variation exactly cancels the integrals in (A.20) and we get $\delta(S + S_{\texttt{bndy}}) = [0]_{\texttt{on-shell}}$. We get

$$
\begin{aligned}
\mathcal{S}_{\texttt{bndy}} &= \mathcal{S}_{\texttt{bndy}}[\psi,\bar{\psi}] + \mathcal{S}_{\texttt{bndy}}[A] \\
\mathcal{S}_{\texttt{bndy}}[\psi,\bar{\psi}] &= -\frac{ia_L}{2}\int d^2\vec{z}\ \left(\bar{\psi}_z^{(1)}(x^0,\vec{z})\psi_{\bar{z}}^{(1)}(x^0,\vec{z}) + \bar{\psi}_{\bar{z}}^{(2)}(x^0,\vec{z})\psi_z^{(2)}(x^0,\vec{z})\right)\Big|_{x^0=0} \\
\mathcal{S}_{\texttt{bndy}}[A] &= a_L\int d^2\vec{z}\ A_z(\vec{z},x^0)A_{\bar{z}}(\vec{z},x^0)|_{x^0=0}
\end{aligned}
\tag{A.23}
$$

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5. Now we will put the solutions obtained in (A.19) in the bulk as well as boundary action and obtain the on-shell action as a functional of the boundary values. The contribution from the boundary action is straightforward to obtain. We get

$$\begin{aligned}\mathcal{S}_{\texttt{bndy}}[\psi,\bar\psi] &= -\frac{2ia_L}{\pi}\int d^2\vec z\, d^2\vec w\, \frac{1}{(z-w)^3}\, \Theta^{(+)}_{\bar z}(\vec w)\, \Theta^{(-)}_{\bar z}(\vec z) \\ &\quad -\frac{a_L}{\pi^2}\int d^2\vec z\, d^2\vec w\, d^2\vec v\, \frac{1}{(z-w)(z-v)(w-v)^2}\, A^{(0)}_{\bar z}(\vec z)\, \Theta^{(+)}_{\bar z}(\vec v)\, \Theta^{(-)}_{\bar z}(\vec w) \\ \mathcal{S}_{\texttt{bndy}}[A] &= -\frac{a_L}{\pi}\int d^2\ \vec z\, d^2\vec w\, \frac{1}{(z-w)^2}\, A^{(0)}_{\bar z}(\vec w)\, A^{(0)}_{\bar z}(\vec z) \\ &\quad -\frac{a_L}{4\pi^2}\int d^2\vec z\, d^2\vec w\, d^2\vec v\, \frac{1}{(z-w)^2(w-v)^2}\, A^{(0)}_{\bar z}(\vec z)\, \Theta^{(+)}_{\bar z}(\vec v)\, \Theta^{(-)}_{\bar z}(\vec w) \\ &\quad -\frac{a_L}{2\pi^2}\int d^2\vec z\, d^2\vec w\, d^2\vec v\, \frac{1}{(z-w)^3(z-v)}\, A^{(0)}_{\bar z}(\vec z)\, \Theta^{(+)}_{\bar z}(\vec v)\, \Theta^{(-)}_{\bar z}(\vec w)\end{aligned} \tag{A.24}$$

In order to obtain the contribution from the bulk action (5.1), we recall from section 5 that the first two terms in the action will not contribute towards the evaluation of two and three point functions. The last term vanish by equations of motion. So the only term that contributes is

$$\begin{aligned}\mathcal{S}_{\texttt{bulk}} &= i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP} A_M\partial_N A_P \\ &= i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP} A^0_M\partial_N A^0_P + i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP} A^0_M\partial_N A^1_P \\ &\quad + i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP} A^1_M\partial_N A^0_P + i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP} A^1_M\partial_N A^1_P\end{aligned} \tag{A.25}$$

In the above equation, we have broken the full solution to a sum of the leading solution and the first order correction. Since $A^0_M = \partial_M\rho$, this implies $\epsilon^{MNP}\partial_N A^0_P = 0$. The last term in (A.25) gives a quartic contribution and hence can be neglected. Thus

$$\begin{aligned}\mathcal{S}_{\texttt{bulk}}[A,\psi,\bar\psi] &= i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP}\ A^0_M\ \partial_N A^1_P \\ &= i\frac{a_L}{2}\int d^3x\, \epsilon^{MNP}\ \partial_N\left(A^0_M\ A^1_P\right) \\ &= a_L\int d^2\vec z\ \left(A^0_z(\vec z)\ A^1_{\bar z}(\vec z) - A^0_{\bar z}(\vec z)\ A^1_z(\vec z)\right)\big|_{x^0=0}\end{aligned} \tag{A.26}$$

In order to evaluate the above contribution from the bulk action we need to know the leading order solution and first order correction separately for the gauge field A_M.

$$\begin{aligned}A^0_{\bar z} &= \partial_{\bar z}\rho \\ &= A^{(0)}_{\bar z}(\vec z) + \frac{1}{4\pi}\int d^2\vec w \frac{1}{(z-w)^2}\ \Theta^{(+)}_{\bar z}(\vec w)\ \Theta^{(-)}_{\bar z}(\vec z), \\ A^0_z &= \partial_z\rho \\ &= -\frac{1}{\pi}\int d^2\vec w\ \frac{1}{(z-w)^2}\ A^{(0)}_{\bar z}(\vec w) \\ &\quad -\frac{1}{4\pi^2}\int d^2\vec v\ d^2\vec w \frac{1}{(z-w)^2(w-v)^2}\ \Theta^{(+)}_{\bar z}(\vec v)\ \Theta^{(-)}_{\bar z}(\vec w)\end{aligned}$$

$$
\begin{aligned}
A^1_z &= \frac{1}{4}\bar{\phi}\,(\partial_z \eta + \phi) + \frac{1}{4}\bar{\eta}\partial_z \phi \\
&= -\frac{1}{2\pi^2}\int d^2\vec{v}\; d^2\vec{w}\; \frac{1}{(z-w)^3(z-v)}\; \Theta^{(+)}_{\bar{z}}(\vec{v})\; \Theta^{(-)}_{\bar{z}}(\vec{w}) \\
A^1_{\bar{z}} &= \frac{1}{4}\bar{\phi}\partial_{\bar{z}}\eta + \frac{1}{4}\bar{\eta}\partial_{\bar{z}}\phi \\
&= -\frac{1}{4\pi}\int d^2\vec{w}\; \frac{1}{(z-w)^2}\; \Theta^{(+)}_{\bar{z}}(\vec{w})\; \Theta^{(-)}_{\bar{z}}(\vec{z})
\end{aligned} \tag{A.27}
$$

Putting these solutions in (A.26), we get the contribution from the bulk action upto terms cubic in the boundary values as

$$
\begin{aligned}
\mathcal{S}_{\texttt{bulk}}[A,\psi,\bar{\psi}] &= a_L \int d^2\vec{z}\; \left(A^0_z(\vec{z})\; A^1_{\bar{z}}(\vec{z}) - A^0_{\bar{z}}(\vec{z})\; A^1_z(\vec{z})\right) \qquad \text{(A.28)} \\
&= \frac{a_L}{4\pi^2}\int d^2\vec{z}d^2\vec{v}d^2\vec{w}\; \frac{1}{(z-w)^2(w-v)^2}\; A^{(0)}_{\bar{z}}(\vec{z})\; \Theta^{(+)}_{\bar{z}}(\vec{v})\; \Theta^{(-)}_{\bar{z}}(\vec{w}) \\
&\quad + \frac{a_L}{2\pi^2}\int d^2\vec{z}d^2\vec{v}d^2\vec{w}\; \frac{1}{(z-v)(z-w)^3}\; A^{(0)}_{\bar{z}}(\vec{z})\; \Theta^{(+)}_{\bar{z}}(\vec{v})\; \Theta^{(-)}_{\bar{z}}(\vec{w})
\end{aligned}
$$

Combining this with the boundary contributions obtained in (A.24), we get

$$
\begin{aligned}
\mathcal{S}[A,\psi,\bar{\psi}] &= \mathcal{S}_{\texttt{bulk}}[A,\psi,\bar{\psi}] + \mathcal{S}_{\texttt{bndy}}[\psi,\bar{\psi}] + S_{\texttt{bndy}}[A] \qquad \text{(A.29)} \\
&= -\frac{a_L}{\pi}\int d^2\; \vec{z}\; d^2\vec{w}\; \frac{1}{(z-w)^2}\; A^{(0)}_{\bar{z}}(\vec{w})\; A^{(0)}_{\bar{z}}(\vec{z}) \\
&\quad - \frac{2ia_L}{\pi}\int d^2\vec{z}\; d^2\vec{w}\; \frac{1}{(z-w)^3}\; \Theta^{(+)}_{\bar{z}}(\vec{w})\; \Theta^{(-)}_{\bar{z}}(\vec{z}) \\
&\quad - \frac{a_L}{\pi^2}\int d^2\vec{z}\; d^2\vec{w}\; d^2\vec{v}\; \frac{1}{(z-w)(z-v)(w-v)^2}\; A^{(0)}_{\bar{z}}(\vec{z})\; \Theta^{(+)}_{\bar{z}}(\vec{v})\; \Theta^{(-)}_{\bar{z}}(\vec{w})
\end{aligned}
$$

Obtaining the correlation functions from here on is straightforward and has been obtained in section 5.

In order to arrive at the solutions of the fields (A.19) in terms of the boundary values, we had to carry out a number of non-trivial "p" integrals. We conclude this section by outlining one of such integrals. The other integrals can be similarly worked out. One of such integrals is

$$
I_2(\vec{z}-\vec{w}) \equiv -\frac{i}{4\pi^2}\int\; d^2\vec{p}\; \left(\frac{p^2_z}{p_{\bar{z}}}\right) e^{i\vec{p}.(\vec{z}-\vec{w})} \tag{A.30}
$$

In order to evaluate the above integral, we define

$$
\begin{aligned}
\Upsilon &\equiv p_z(z-w) \\
\bar{\Upsilon} &\equiv p_{\bar{z}}(\bar{z}-\bar{w}) \\
\Rightarrow \qquad d^2\vec{p} \equiv -2idp_z dp_{\bar{z}} &= \frac{-2i}{(z-w)(\bar{z}-\bar{w})} d\Upsilon d\bar{\Upsilon} \\
&\equiv \frac{1}{(z-w)(\bar{z}-\bar{w})} d^2\vec{\Upsilon}
\end{aligned} \tag{A.31}
$$

$d^2\vec{\Upsilon}$, similarly to $d^2\vec{p}$ in (A.10) and (A.11), can be expressed in terms of its real and imaginary parts

$$d^2\vec{\Upsilon} \equiv d\Upsilon_1 d\Upsilon_2$$
$$\texttt{where} \qquad \Upsilon_1 \equiv \frac{1}{2}\texttt{Re}(\Upsilon), \quad \Upsilon_2 \equiv -\frac{1}{2}\texttt{Im}(\Upsilon), \tag{A.32}$$

With all the above definitions, the integral (A.30) takes the following form

$$\begin{aligned} I_2(\vec{z}-\vec{w}) &= -\frac{i}{4\pi^2}\frac{1}{(z-w)^3}\int d^2\vec{\Upsilon}\ \left(\frac{\Upsilon^2}{\bar{\Upsilon}}\right) e^{i(\Upsilon+\bar{\Upsilon})} \\ &\equiv \frac{c_2}{(z-w)^3} \end{aligned} \tag{A.33}$$

The last line of the above equation defines c_2 as

$$c_2 \equiv -\frac{i}{4\pi^2}\int d^2\vec{\Upsilon}\ \left(\frac{\Upsilon^2}{\bar{\Upsilon}}\right) e^{i(\Upsilon+\bar{\Upsilon})} \tag{A.34}$$

Thus the "p" integral (A.30) now boils down to doing the "Υ" integral (A.34). In order to do the integral (A.34) we define

$$\begin{aligned} & \Upsilon_1 = R\cos\theta, \qquad \Upsilon_2 = R\sin\theta \\ \Rightarrow \qquad & d^2\vec{\Upsilon} = R\ dR\ d\theta, \quad \Upsilon = \frac{1}{2}Re^{-i\theta}, \quad \bar{\Upsilon} = \frac{1}{2}Re^{i\theta}, \quad \Upsilon+\bar{\Upsilon} = R\cos\theta \end{aligned} \tag{A.35}$$

Putting this definition in (A.34), we get

$$\begin{aligned} c_2 &= -\frac{i}{4\pi^2}\frac{1}{2}\int_0^\infty R^2\ dR\int_0^{2\pi} e^{i(Rcos\theta-3\theta)}d\theta, \\ &= -\frac{1}{4\pi}\int_0^\infty R^2 J_3(R)\ dR = -\frac{2}{\pi} \end{aligned} \tag{A.36}$$

In the last step, we have used the integral representation of Bessel's function

$$J_n(x) = \frac{i^{-n}}{2\pi}\int_0^{2\pi} e^{i(x\cos\theta+n\theta)}\ d\theta \tag{A.37}$$

And

$$\int_0^\infty J_n(R)\ dR = 1 \qquad \int_0^\infty RJ_{n-1}(R)\ dR = n-1 \qquad \int_0^\infty R^2 J_{n-1}(R)\ dR = n(n-2) \tag{A.38}$$

Therefore the integral (A.30) takes the form

$$I_2(\vec{z}-\vec{w}) = -\frac{2}{\pi(z-w)^3} \tag{A.39}$$

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